

A MATHEMATICAL MODEL FOR ALLOCATION OF
SCHOOL RESOURCES TO OPTIMIZE
A SELECTED OUTPUT

By

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Abstract of Dissertation Presented to the Graduate Council
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A MATHEMATICAL MODEL FOR ALLOCATION OF
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Educational decision-makers are faced with the task of allocating school resources in such a way as to maximize student outcomes. Decision-makers are constrained in their choice of plans by a limited budget and by social, political, and legal forces. The rationale for this study was a response to the need for studies that demonstrate the utility and limitations of mathematical models as aids in improving educational planning and decision-making.

The problem of the study was to develop a mathematical model to facilitate the decision-making process in selected areas of educational activity by optimizing the allocation of scarce resources and to empirically illustrate its application. In development of the model attention was given to

1. Determining the educational production function that describes the input-output relationship between selected variables;
2. Determining the optimal combination of inputs to maximize output subject to certain costs and other constraints;
3. Determining the optimal combination of inputs to maximize output, subject to certain constraints, given increments in the budget constraint.

In the empirical application, data were utilized to illustrate the model. The data consisted of aggregate measures on input and output factors in 181 local school districts in a given state. Twenty-four variables were measured providing information on (1) student and community inputs, (2) median reading achievement at the sixth-grade level, and (3) selected school resources. In particular, certain teacher characteristics were considered: experience, training, and level of salary. Also, classroom size, pupil-support personnel ratio, and the size of the district were used.

Given these inputs (i.e., student, community, and school) into the educational process, stepwise multiple regression was used to estimate the parameters of the production function (reading achievement--dependent variable)

and the budget equation (per-pupil instructional expenditures--dependent variable). The proportion of variance explained in the dependent variables was 0.79 and 0.94, respectively.

The optimization strategy employed the techniques of mathematical programming with the production function as an objective function and the budget equation as one of the constraints. Specifically, the SUMT (Sequential Unconstrained Minimization Technique) algorithm was employed to solve the mathematical programming problems. Looking at the population of school districts as a whole, the analysis revealed that by reallocating existing school resources and maintaining a minimum standard for the pupil-support personnel ratio, the predicted increase in reading achievement that would result was 0.38 standard deviations. Given the existing allocation of school resources, if the per-pupil budget was incremented by \$100 so as to maximize reading achievement, the predicted increase would be 0.73 standard deviations.

Throughout the analysis it was apparent that the percentage of teachers with advanced training was the dominant effect followed by median teacher salary. The importance of salary was a function of the percentage of teachers with less than four years' training; the greater the percentage,

the greater the effect of salary. When the percentage was zero, the effect of salary was minor compared to the effect of advanced training.

CHAPTER I

INTRODUCTION

For too long the United States has tolerated a system of compulsory universal education without pertinent public accountability. As a result critical deficiencies are now becoming evident--such as the failure to serve inner-city students effectively. . . . They (public) have a right to know how well students are learning as well as a responsibility to pay for the educational program.¹

Across the land, state legislatures have sounded the clarion call of "accountability" for educational institutions. Politicians have asked for specific and quantitative information on the schools' output. Educators' requests for increased educational expenditures have been matched by legislators' demands for increased productivity.

Concurrently, educational institutions have begun efforts to respond to the demands of new technologies. Experimentation with the concepts of systems analysis and operations research have occurred in an attempt to place

¹L. J. Stiles, "Assessment of Educational Productivity," Journal of Educational Research, LX (April, 1967), 1.

decision-making activities on a more rational plane. The Planning-Programming-Budgeting-System (PPBS) found its place in the public sector, first at the federal level and then in state and local governmental agencies.² "Attention is focused on choosing from many possible objectives those specific objectives to be achieved, and then choosing from alternative courses of action that plan which will accomplish the chosen objectives at the lowest possible cost, or accomplish some more optimum set of objectives at a specified cost."³ For school purposes, the core of PPBS is the program budget that reports programs to be accomplished and allocates expenditures in terms of objectives relating to student outputs rather than in terms of objects to be purchased. Two outcomes of a PPB system are cost-effectiveness and cost-benefits analyses. Cost-effectiveness is concerned with the extent to which given dollar inputs if utilized in alternative ways would increase certain educational outputs. The main requirement of cost-benefits

²Marvin C. Alkin, "Evaluating the Cost-Effectiveness of Instructional Programs," UCLA Symposium on Problems in the Evaluation of Instruction, Sponsored by the Center for the Study of Evaluation (Los Angeles: December, 1967).

³H. Thomas James, The New Cult of Efficiency and Education (Pittsburgh: University of Pittsburgh Press, 1969), p. 40.

analysis is that both input and output measures be specified in the same unit, namely dollars; then the ratio of benefits to costs for a given program is examined.⁴

With new developments in technology, economists and educational researchers have increased their efforts in the study of the internal efficiency of education. These studies require measurements of both the inputs and outputs of educational systems--a difficult task. Even more difficult is the recognition that there are too many variables affecting school performance for researchers to experimentally control and manipulate them all at once. "Thus researchers have had to rely on survey-type correlation analyses and infer from them the causes of variation in educational success."⁵ Hence, input-output models from economics have been adapted for the study of productivity in education. This has not been done without experiencing difficulties. Usually in industry, input-output relationships are such that empirical relationships can be determined within a theoretical framework. Such is not the case in education where concern with behavioral characteristics is wanting of both a theoretical and quantitative basis.

⁴Alkin.

⁵Michigan Department of Education, Research into Correlates of School Performance, Assessment Report No. 3 (1970), p. 14.

Numerous input-output studies have taken place in education in recent years; however, most of these have not considered questions of cost-effectiveness. Of the cost-effectiveness studies, few have related their techniques to input-output analysis. Consequently, the focus of this study was the development of a model that integrates both concepts and leads to decision-making on a plane conceived by the advocates of PPBS. The demands of the age are such that if the educational enterprise is to survive, progress must be made towards the development of a theory of decision-making that effectively relates the inputs of a school system to its outputs.

The Problem

Statement of the Problem

The problem of the study was to develop a mathematical model to facilitate the decision-making process in selected areas of educational activity by optimizing the allocation of resources and to empirically illustrate its application. In development of the model attention was given to

1. Determining the production function that describes the input-output relationship between selected variables;
2. Determining the optimal combination of inputs to maximize output subject to certain costs and other constraints;

3. Determining the optimal combination of inputs to maximize output, subject to certain constraints, given increments in the budget constraint.

In the empirical application, data obtained from the National Educational Finance Project were utilized to illustrate the model. On the basis of these data an educational production function was estimated and optimal allocations of resources were determined subject to certain constraints.

Delimitations

The study was confined as follows:

1. To educational outcomes of a cognitive nature, that is, behaviors dealing with the recall or recognition of knowledge and development of intellectual abilities and skills;

2. To the investigation of allocative efficiency; using the dollar budget in such a way that the resources that are purchased yield the best outcome that can be attained for the given budget (i.e., to identify feasible ways of improving output within the context of the present system by using the components presently employed in the system);

3. To an investigation of school districts within a given state in the empirical application;
4. To an investigation of aggregate educational policy (what is true for the population as a whole may not be true for individual school districts).

Limitations

The study was subject to the following limitations:

1. The use of unidimensional outcomes;
2. The explanatory power of the particular input variables selected and their usefulness from a decision-making context;
3. The problems associated with imprecise measurement of the variables selected;
4. The use of a developmental design and the limitations inherent in determining a mathematical model:
 - a. the lack of a theoretical basis for postulating a specific model;
 - b. the necessity for a statistical dimension and the resulting restrictions on the complexity of the model design;
 - c. because social behavior is less predictable than physical phenomena, mathematical models are a less accurate approximation of reality in the social sciences;

5. The use of survey data and the inability to attach causality to estimated parameters.

Justification for the Study

Since the most commonly used tool for measuring economic relationships is multiple regression analysis, it is no surprise that as researchers have examined the internal efficiency of education, regression analysis has been the primary tool.⁶ Also, the economists' stress on regression coefficients rather than the proportion of variance explained is a new development in educational research.⁷ As the review of the literature describes, educational production functions have had too many flaws to be effectively used in the past.⁸ Nevertheless, an assessment of the optimality of resource allocation in education requires a knowledge of the production process.⁹ Benson, in considering

⁶Arthur S. Goldberger, Topics in Regression Analysis (New York: Macmillan, 1968), p. 1.

⁷Mary Jane Bowman, "Economics of Education," Review of Educational Research, XXXIX (December, 1969), 642.

⁸Elchanan Cohn, "Towards Rational Decision-Making in Secondary Education," Institute for Research on Human Resources (University Park: Pennsylvania State University, 1970), p. 8.

⁹Ibid., p. 2.

resource allocation in the education process to attain objectives, has stated that the following conditions must be met: (1) that objectives can be precisely stated; (2) units in which the objectives are achieved can be defined; (3) variables which influence the achievement of an objective can be identified and that units of these variables can be defined and priced; and (4) the relationships between input variables and objectives can be specified.¹⁰ Bowman has stated, "Proper identification of production functions is indeed necessary for sound cost-effectiveness or cost-benefits analysis in educational decision making."¹¹

Bowles has stated, "Without knowledge of the educational production process, we are unable to estimate the relative effectiveness of each factor input per unit of cost, nor are we able to compare the relative productivity of resources devoted to different types of education or non-educational purposes."¹² Entwistle and Conviser noted that

¹⁰Charles S. Benson, The School and the Economical System (Chicago: Science Research Associates, 1966), p. 102.

¹¹Bowman, p. 652.

¹²Samuel Bowles and Henry M. Levin, "More on Multicollinearity and the Effectiveness of Schools," Journal of Human Resources, III (Summer, 1968), 28.

input-output analysis deals only with the possibility of certain states of affairs. It is not suitable for making determinations as to the most desirable state of affairs, such as producing certain outputs at minimum costs. "There has developed a more general approach known as linear programming, which is appropriate for optimization problems, and input-output analysis turns out to be a special case of linear programming."¹³ Bowles stated that "the task of statistical and educational researchers should be directed towards estimating the structural parameters of an equation representing a learning process."¹⁴

Thus, if efficient resource allocation is to be more than a figure of speech, the previous statements must be considered. To the extent that a more effective marshaling of school resources maximizes student outputs, the attainment of the goals of education are enhanced and support for proper financing is generated. Hence, the study was justifiable for the following reasons:

¹³Doris R. Entwisle and Richard Conviser, "Input-Output Analysis in Education," High School Journal, LII (January, 1969), 194.

¹⁴Samuel Bowles, Planning Educational Systems for Economic Growth (Cambridge, Mass.: Harvard University Press, 1969), p. 400.

1. Refinements in the techniques that develop production functions are needed if a meaningful analysis is to occur;

2. Appropriate optimization techniques must be developed if the goals of resource allocation are to be met;

3. Effective interfacing between the production function and the budget equation must result if cost-effectiveness analysis is to be realized.

Assumptions

1. It was assumed that instruments used to measure inputs and outputs in the model were valid.

2. It was assumed that cost per unit of input could be determined for certain inputs used in the model.

3. It was assumed that a relationship existed between the inputs and outputs used in the model.

Definition of Terms

Budget equation.--The relationship between the dollar budget accorded educational decision-makers and the transformation of that budget into school inputs.

Cost-effectiveness analysis.--A procedure by which the costs of alternative means of achieving a stated effectiveness or, conversely, the effectiveness of alternative means

for a given cost, are compared to determine that alternative or combination of alternatives that either gives the greatest expected effectiveness for a given expected cost or a given expected effectiveness for the least expected cost.

Constraint.--Any procedure, practice or condition that limits the manner in which an activity or decision-making is carried out.

Decision-making.--The series of actions and interactions through which questions or issues are resolved or disposed.

Interfacing.--The process of using certain variables measured in the same units in both the production function and the budget equation for the purpose of optimization.

Mathematical programming.--A systematic method for finding maximum and minimum values of mathematical functions subject to certain constraints.

Marginal effect.--The increase in output which can be obtained by a unit increase in an input.

Multiple regression analysis.--The process of estimating a variable y from the knowledge of several predictor variables $x_1, x_2, \dots, x_n$.

Optimization.--The best or most favorable amounts of inputs for the purposes of maximizing outputs or minimizing costs.

Parameter.--A measure computed from all observations in a population.

Production function.--The relationship between an outcome of education and the human and material resources which education consumes.

Resource allocation.--The process of assigning or allocating resources among the various components of an educational system.

Review of the Literature

The review summarizes past studies which were pertinent to the purposes of the present study. The main areas of concern were

1. Input-output analysis,
2. Cost-effectiveness analysis,
3. Methodological concerns.

Since methodological considerations were of prime importance, emphasis was focused on this aspect.

Input-Output Analysis

In an attempt to determine factors that are related to school performance, educators have employed various models or research paradigms. Input-output models have their origins in the work of Leontief in economics. Leontief conceptualized a transactions table for displaying relationships between economic inputs and outputs for purposes of budget analysis and planning.¹⁵ Hoffenberg applied this model to a school district budget. The assumption was that the activities which a school district carries on are interrelated, hence Hoffenberg determined the structural framework of these relationships.¹⁶ The goal of such input-output studies is to obtain estimates of the marginal effects of an increase in a given input variable.

Researchers have adapted the input-output model of Leontief in an attempt to account for the causes of variation in school performance. "Researchers employing this

¹⁵ Marvin C. Alkin, "The Use of Quantitative Methods as an Aid to Decision-Making in Educational Administration," A paper read at a meeting of the American Educational Research Association (Los Angeles: 1969).

¹⁶ Marvin Hoffenberg, "Application of Leontief Input-Output Analysis to School District Budgeting," Center for the Study of Evaluation: Working Paper No. 12 (Los Angeles: UCLA Graduate School of Education, 1970).

paradigm have: (1) identified a criterion of school performance as a dependent variable and measures thought to influence performance as independent variables; (2) operationally measured these variables in a sample of educational systems; (3) computed relationships between independent and dependent variables; and (4) drawn inferences from the relationships as to what factors, either singly or in combination, account for variation in school performance."¹⁷

The mathematical relationship between inputs and an output is termed a production function. Outputs, which are behavioral changes in students, and their relationships to inputs are termed psychological production functions.¹⁸ This function seeks to relate performance increments to inputs.

Three types of studies exist which seek to determine the relationships between school inputs and educational outcomes.

The first type of study attempts to link total educational expenditures to output measures. The second group includes those that have attempted to estimate the effects of various functional

¹⁷Michigan Department of Education, p. 1.

¹⁸J. Alan Thomas, The Productive School: A Systems Analysis Approach to Educational Administration (New York: Wiley & Sons, Inc., 1971), p. 12.

components of expenditure on school outcomes, and the third set of studies represents estimates of the relationships between resources as measured in physical terms and school outputs (educational production functions).¹⁹

For studies of the first type, emphasis was placed on the financial side of the ledger (i.e., more money does more things) without concurrence as to what is quality.²⁰

Actually, quality of education was seen to be a measure of school practices. For studies of the second type, emphasis was placed on determining which group of resources, that the school purchases, gives the highest educational returns per dollar input. A problem was that these studies did not address themselves to how the resources were being used. Given that one has purchased a particular combination of physical resources, there are alternative ways of utilizing these resources to affect output.²¹

With the advent of systems analysis in industry and the concern with the nature of output, studies of the third

¹⁹Henry M. Levin, "The Effect of Different Levels of Expenditure on Educational Output," Economic Factors Affecting the Financing of Education (Gainesville, Florida: National Educational Finance Project, 1970), II, 185.

²⁰Michigan Department of Education, pp. 1, 2.

²¹Levin, pp. 182, 188.

type have emerged. Kershaw and McKean pioneered the application of systems analysis to education. "This work outlined persuasively and clearly the values of treating education as a system amenable to quantitative analysis, capable of evaluation in terms of efficiency, and responsive to policy decisions regarding inputs and administrative arrangements."²² After this several input-output studies followed that sought to relate various measures of student performance to school and community characteristics. Thomas, Benson, Coleman, Burkhead, and Kiesling sought to determine the factors most related to achievement. In each of these studies, the most highly related characteristics were found to be socio-economic.²³ Most of these studies used regression methods to determine the production functions. Criticisms of this methodology are discussed in a later section.

Cost-effectiveness Studies

It is assumed that resources should be allocated so as to maximize the achievement of a given

²²Jerry Miner, "Financial Support of Education," Implications for Education of Prospective Changes in Society, ed. by Edgar L. Morphet and Charles O. Ryan (Denver: Designing Education for the Future, an Eight-State Study, 1967), p. 313.

²³Michigan Department of Education, p. 15.

objective or set of objectives. Cost analysis provides a means for examining the manner in which resources are used.²⁴

Temkin stated that cost-effectiveness methods are appropriate when the decision structure is suitable for planned and incremental improvement of an existing system with a time dimension weighted heavily towards the present and immediate future.²⁵ Thomas described five elements of cost-effectiveness analysis: (1) the objective, (2) the alternatives, (3) costs, (4) operational model, and (5) a decision rule.²⁶ If the operational model is the production function described earlier, then one should be able to estimate the allocative pattern among inputs that will yield the largest increase in output within a limited budget. Few such studies have been carried out for educational productivity, in part, because the 'prices' of such inputs are difficult to derive.²⁷ As a result, most input-output models used to date have not included costs.²⁸

²⁴Thomas, p. 41.

²⁵Sanford Temkin, "A Comprehensive Theory of Cost-Effectiveness: Administration for Change Program" (Philadelphia: Research for Better Schools, Inc., April, 1970), p. 41.

²⁶Thomas, p. 82.

²⁷Levin, p. 191.

²⁸Thomas, p. 17.

One exception is Hoffenberg, who applied an interdependency approach in an input-output framework to a school district budget. Using legal program descriptions he was able to determine

1. interprogram flows--the input distributions contributing to output,
2. direct purchases per dollar of output (what is required from each activity to give \$1 of output to a designated activity),
3. direct and indirect requirements per \$1 of final output from each of the activities.²⁹

It must be noted, however, that the outputs of Hoffenberg's study were school programs rather than student outputs. Nevertheless, where psychological production functions were involved, Thomas stated that input-output relationships could be measured in terms of costs per student per unit of time for a given input. As an example, to determine the direct cost per student hour of a given service would require cost figures for (1) teachers' salaries, (2) other salaries of personnel who support the service, (3) space, and (4) equipment and materials.³⁰

In designing a cost-effectiveness model for elementary and secondary education, Abt made the following recommendations:

²⁹Hoffenberg, p. 6.

³⁰Michigan Department of Education, p. 50.

1. The relative contributions of home, peer influence, and school instruction to student achievement should be determined,
2. The relative contributions of manipulable variables in the school environment which contribute most heavily to student attitude and achievement change should be determined,
3. The coefficients and parameter settings for environmental influences on student attitude and achievement change should be determined with enough accuracy to allow useful predictions.³¹

Cohn discussed the use of linear programming as a means of optimizing outputs. He regarded existing production functions as being incomplete. Hence, he used sub-optimization techniques where he varied at most three variables simultaneously. Consequently, the overall results were less than optimal.³² The New York State Education Department studied the feasibility of using state department data to develop multiple regression equations required for simulating the effects of alternative expenditure configurations on pupil achievement. Various

³¹Clark C. Abt, Design for an Elementary and Secondary Education Cost-Effectiveness Model (Cambridge, Mass.: Abt Associates, Inc., 1967), II, 142.

³²Cohn.

configurations of input were considered to maximize output, but optimization techniques were not discussed.³³

Methodological Considerations

For most production processes, an increase in the quantity of an input is expected to yield an increase in output. However, Ribich stated,

Observation of the effect may be difficult because the additional resource input is small relative to other changes that are occurring simultaneously. The "upshot" can be difficulty in statistically sorting out the independent influence of the additional inputs from the other influences at work, many of which are unmeasured and unmeasurable. The problem may be so severe as to submerge almost completely any evidence of output response to an input change.³⁴

Wynne stated that most input-output studies in education have used the techniques of multiple regression analysis.³⁵ Consequently, as a result of Ribich's previous statement, the use of multiple regression analysis must be examined with particular attention to interpretation of results. A brief description of the method follows.

³³New York State Education Department, "Technical Report of a Project to Develop Education Cost-Effectiveness Models for New York State" (Albany: Bureau of School Progress and Evaluation, March, 1970).

³⁴Thomas I. Ribich, "The Effect of Educational Spending on Poverty Reduction," Economic Factors Affecting the Financing of Education, II, 214.

³⁵Edward Wynne, "School Output Measures as Tools for Change," Education and Urban Society, XII (November, 1969).

The process whereby performance on a single-criterion variable is predicted from a knowledge of several predictor variables is called multiple regression. Predictors, undoubtedly, will differ in their predictive efficiency. Therefore, computational processes are directed towards determining a separate regression weight (partial regression coefficient) for each predictor, in order to achieve the best possible prediction. The result may be a regression equation of the form:

$$\hat{y} = b_0 + b_1x_1 + b_2x_2 + \dots + b_nx_n$$

where y = criterion (dependent) variable, $\hat{y}$ = predicted value, and x_i = predictor (independent) variable. A measure of overall efficiency in prediction is given by R (multiple correlation coefficient), which is the correlation of predicted values $\hat{y}$ to actual values y . More importantly, R^2 gives us the proportion of variance of y accounted for by the regression relationship.³⁶

The partial regression coefficient b_i , estimates the marginal effect upon y with all other x 's held constant.³⁷ However, if the sample variation of x_i is small, the

³⁶Goldberger.

³⁷Ibid., p. 26.

imprecision which attaches to its coefficient will tend to be large.³⁸ If the chief concern is with the marginal effect, then a high correlation is needed between y and x_i , or the confidence limits on B_i (parameter estimated by b_i) will be too wide.³⁹ Next, the reliability of the instruments used to measure the x_i must be considered. Regression coefficients b_i will tend to overestimate B_i with less than perfect reliability of the instruments used.⁴⁰

But of all of the problems discussed thus far, the most serious is the problem of multicollinearity. Multicollinearity arises when some or all of the explanatory variables are so highly correlated one with another that it becomes very difficult, if not impossible, to disentangle their influences and obtain a reasonably precise estimate of their separate effects.⁴¹ Gordon stated that small variations among the correlations of a highly related set

³⁸Ibid., p. 60.

³⁹Phillip Lyle, Regression Analysis of Production Costs and Factory Operations (New York: Hafner Publishing Co., Inc., 1957), p. 65.

⁴⁰George W. Bohrnstedt, "Observations on the Measurement of Change," Sociological Methodology, ed. by Edgar F. Borgatta (San Francisco: Jossey-Bass, Inc., 1969), p. 128.

⁴¹Goldberger, p. 80.

of independent variables can create large variations among their regression coefficients.⁴² A standard approach to this problem has been to utilize additional information as an aid in estimation. Knowledge of the ratios of regression coefficients from theory or from previous samples can be used. Even incorrect information can reduce the imprecision of estimation by using Bayesian techniques.⁴³ Finally, one may ask if β_i (beta weight-standardized regression coefficient) is not a more effective statistic. Bohrnstedt stated that the regression coefficient b_i is more appropriate for the study of change than β_i , since b_i is relatively stable across subsamples of a population where β_i may vary significantly as a function of the standard deviations of the variables in the sample.⁴⁴

The application of regression theory in estimating educational production functions has been discussed in the literature. In attempting to interface the production function to the budget equation, particular care must be placed on the interpretation of regression coefficients.

⁴²Robert A. Gordon, "Issues in Multiple Regression," American Journal of Sociology, LXXIII (March, 1968), 162.

⁴³Goldberger, p. 82.

⁴⁴Bohrnstedt, p. 120.

The other aspect of regression analysis is the proportion of accounted variance. The Coleman report has been soundly criticized for the manner in which it sought to identify important variables on the basis of their contributions to the porportion of variability explained. As Bowles and Levin stated, using stepwise procedures, the relative contributions to variability are dependent upon the order in which the independent variables are entered into the equation. They suggested that Coleman should have used the regression coefficients.⁴⁵ Even though this is a more effective procedure, if multicollinearity is severe, the regression coefficients may be woefully inaccurate.

Additional problems in interpretation are noted. Levin suggested that output measures should be value-added, that is, the change that has occurred while in school.⁴⁶ Borhnstedt raised serious questions involving the use of gain scores in measuring change.⁴⁷ Nephew, in constructing an input-output model, "partialed out" the variance due to non-school variables and then worked with residual measures

⁴⁵Bowles and Levin.

⁴⁶Levin, p. 176.

⁴⁷Bohrnstedt.

of school output using multiple regression equations.⁴⁸ Goldberger showed that regressing on the residuals will underestimate the regression coefficients of the remaining variables.⁴⁹

Bowles in a comprehensive study of the educational production function found that the function generally explains only a small percentage of the variability of school achievement using the full range of variables. A crucial deficiency is the absence of a theory of the learning process. A conclusion was that it would be useful to think of distinct educational production technologies at least for black and white and rich and poor students separately. Also, due to "ceiling effects" it would be more appropriate to work with ratios of regression coefficients rather than absolute magnitudes.⁵⁰ This is related to Levin's assertion that the law of diminishing marginal returns is operative in the production function, that is, as more and more

⁴⁸Charles T. Nephew, Guides for the Allocation of School District Financial Resources, Ed.D. Dissertation (Buffalo: State University of New York, 1969).

⁴⁹Arthur S. Goldberger, "Note on Stepwise Least Squares," Journal of the American Statistical Association, LVI (March, 1961).

⁵⁰Samuel Bowles, "Educational Production Function: Final Report," Research Project Supported by the United States Office of Education (Harvard University, February, 1969), pp. 5, 13, 18.

of a given input is consumed its marginal effect decreases.⁵¹

In an attempt to refine techniques that have been used, factor analysis has been tried to meaningfully group the variables to avoid the problem of multicollinearity. The results were disappointing since variables grouped under statistical considerations were not necessarily meaningful from a decision-making context.⁵² Wynne suggested the use of high and low productive schools to identify factors that discriminate between them on the basis of productivity.⁵³ Kiesling stated that it is difficult to determine an accurate production function since one cannot easily gather data in units proper to each variable.⁵⁴

While considerable research has taken place in the study of input-output relationships, less has been done in the interfacing of the production function and the budget equation, as conceived by Levin, for purposes of optimizing

⁵¹Levin, p. 177.

⁵²New York State Education Department.

⁵³Wynne.

⁵⁴Herbert J. Kiesling, "The Study of Cost and Quality of New York School Districts: Final Report," Research supported by the United States Office of Education, Project No. 8-0264 (Bloomington: Indiana University, 1970), p. 66.

outputs subject to budget constraints. As Levin stated, the educational establishment needs a production function and budget relationship to tie educational expenditures to educational outcomes.⁵⁵ Essentially, the concern is that of optimization, but limitations on optimization in an educational context have been expressed. Tracz spoke of the lack of suitable objective functions or performance criteria as strong limitations. Also, education is a non-stationary process which delimits the usefulness of fixed coefficients in objective functions.⁵⁶ Alkin identified impediments to the use of linear programming. First, there is the failure to designate precisely educational programs within educational systems and the attendant financial costs. This is one of the challenges to be met by Planning-Programming-Budgeting Systems (PPBS). Secondly, there is a lack of specificity in the designation of educational outcomes.⁵⁷

⁵⁵Levin, p. 178.

⁵⁶George S. Tracz, "An Overview of Optimal Control Theory Applied to Educational Planning," A paper read at a meeting of the American Educational Research Association (Los Angeles, 1969), p. 5.

⁵⁷Alkin, "The Use of Quantitative Methods as an Aid to Decision-Making in Educational Administration," p. 6.

A more serious limitation to optimization using linear programming is the nature of fixed marginal effects when using a linear production function. If the production function and budget equation were expressed respectively as:

$$Y = b_0 + b_1x_1 + \dots + b_nx_n$$

$$R = P_1x_1 + P_2x_2 + \dots + P_nx_n$$

where R = total budget, and P_i = price per unit of input x_i , then according to Levin, the objective would be to satisfy the condition

$$\frac{\partial y_{x_1}}{P_{x_1}} = \frac{\partial y_{x_2}}{P_{x_2}} = \dots = \frac{\partial y_{x_n}}{P_{x_n}}$$

(i.e., the additional output from each input relative to its price should be equal for all inputs).⁵⁸ However, with constant marginal effects, questions of optimal economies of scale cannot be considered.

Generalizations from the Literature

The review of literature appears to justify the following generalizations.

1. There is a need for decision-making based upon input-output relationships.

⁵⁸Levin, p. 178.

2. The challenge for input-output studies is to determine the proper mix of inputs that will maximize output.
3. In general, input-output studies have not interfaced with cost-effectiveness studies.
4. Serious reservations regarding methodology in input-output studies have been expressed.
5. A mathematical model is appropriate for systematically determining optimal allocation strategies.

Procedures

In the process of developing and applying the mathematical model, attention was given to (1) conceptual considerations, (2) mathematical considerations, and (3) an empirical illustration.

Conceptual Considerations

The process of developing a mathematical model required that one proceed in the presence of a conceptual framework. Consideration was given to a conceptualization of the process of education. The process was viewed in an input-output context. The inputs consisted of community and student factors, and school resources. The outputs were educational outcomes that are desired such as cognitive achievement, affective growth, and physical development.

The relationships between inputs and outputs were termed educational production functions.

As decision-makers seek to effectively allocate resources, their choices are constrained by various social, economic, and legal forces. Thus attention was given to an explicit description of constraints on the process of education.

Efficient resource allocation requires two types of efficiency, technical and allocative. Consideration was given to distinctions between these two types of efficiency and their implications for estimating educational production functions.

Given an understanding of the process of education as revealed by the production function and the various constraints operating on the process, the need for optimization strategies whereby decision-makers can seek to maximize certain outcomes subject to various constraints was deliberated.

Mathematical Considerations

Given a conceptual framework of the process of education, the translation to a mathematical model required consideration of several issues.

Attention was given to estimating single output equations versus multiple output equations. In using multiple regression to estimate the production function, consideration was given to (1) interpretation of regression coefficients, (2) determining goodness of fit, (3) linear versus nonlinear models, (4) reliability of measurements, (5) the use of gain scores, (6) population data versus sample data, (7) causality and multiple regression, and (8) experimental design versus survey design.

Given the need for optimization, the mathematical formulation of an optimization strategy was elaborated. In particular, an algorithm for solving nonlinear programming problems was discussed.

An Empirical Illustration

Given the final form of the mathematical model, steps were taken to empirically illustrate the model. Using real data, an educational production function was estimated. Subject to certain constraints, optimal combinations of inputs were determined to maximize a selected output. Given increments in the budget constraint, the problem was resolved. The exact procedures for the empirical analysis are detailed in Chapter III.

Organization of the Remainder of the Study

In Chapter II the mathematical model is developed and described. In Chapter III an empirical application of the model is demonstrated. In Chapter IV a summary, discussion, and suggestions for future studies are given.

CHAPTER II

DEVELOPMENT AND DESCRIPTION OF THE MATHEMATICAL MODEL

While interest centers on the development of a mathematical model which describes in a systematic fashion a strategy of optimization, it must be remembered that mathematical models per se do not evolve in the absence of a conceptual framework. Antecedent to a mathematical model is a conceptual model which is a verbalization of concepts and ideas that purport to describe the phenomenon under investigation. The attempt to translate the abstractions of a conceptual model into equivalent mathematical statements only accentuates the fact that both are man-made. Thus, the conceptual model serves only as an approximation to reality, and the mathematical model is an approximation to the conceptual model. In this sense, the conceptual model becomes an upper limit for the adequacy of the mathematical model. In short, the mathematical model cannot describe the phenomena under consideration in any better fashion than the conceptual model which precedes it.

The first part of this chapter is devoted to a discussion of conceptual considerations for a mathematical model. Secondly, mathematical considerations implicit in model development are discussed. This provides the basis for stating the methodology which must be employed to make the mathematical model operational. Finally, the complete mathematical model is presented.

Conceptual Considerations

The basic notion of a model pertains to a simplification of reality that allows the user to analyze particular aspects of a process--in this case the production of education. The conceptual framework represents the theoretical backdrop for the mathematical model. Armor has presented a model of aggregate educational processes.¹ The basic structure of this model is discussed with some modifications made to allow a consideration of educational processes at the levels of student, school, district, and state. To formulate a conceptual framework whereby the mathematical model can be developed, it is necessary to consider (1) an educational production function, (2) constraints and the

¹David J. Armor, "School and Family Effects on Black and White Achievement: A Reexamination of the USOE Data," in On Equality of Educational Opportunity, ed. by Frederick Mosteller and Daniel P. Moynihan (New York: Random House, 1971), pp. 171-177.

educational process, (3) technical and allocative efficiency, and (4) the need for optimization.

An Educational Production Function

Any model must begin by deciding what are the appropriate goals of schooling. First and foremost is the learning of basic cognitive skills such as reading, writing, and arithmetic. The mastery of such skills is deemed necessary to acquire a level of competence sufficient to function in a technological society. Secondly, schools offer instruction in a variety of subject fields that enable students to discover interests and aptitudes relevant to future occupational choices. Third, school programs seek to establish positive affective outcomes regarding self-concept, identification with others, openness to experience, and a commitment to social institutions and the culture in which they are enshrined. Finally, schools provide opportunities for aiding the physical and psychological welfare of children with clinics, physical education, school-lunch programs, and guidance services.²

Schools attempt to achieve goals by the use of programs, staff, and facilities. No doubt, the program

²Ibid., p. 171.

determines how the school will be organized, what facilities will be required, and the number and type of staff that will be employed. Each of these factors is potentially capable of manipulation by school authorities. The manner in which these factors are manipulated creates variations across schools and school districts. Concurrently, it may be assumed that these variations create different levels of effectiveness in attaining the goals described above. It is also apparent that the success schools have in working towards these goals is dependent in part upon the background and characteristics which students bring with them to school. This fact has been amply demonstrated in such research as the Equality of Educational Opportunity Survey (EEOS).³

Armor has distinguished several components of the characteristics of students which precede their entry into school and interact with the school environment throughout their school careers. First, there is the environment of the student's family. In the basic unit of society, family life-styles, goals, attitudes, and morals have significant impact on how well the child succeeds in school. Second, the community or neighborhood in which the student lives

³James S. Coleman et al., Equality of Educational Opportunity (Washington, D.C.: U.S. Government Printing Office, 1966).

also helps to define the student. The interpersonal relationships which a student develops with other members of the community, and the manner in which the community manifests itself in facilities, housing, and programs can serve as a debilitating or helping force for the student's growth in school. Finally, there is the peer or student-body factor. The extent of similarity between aggregated student characteristics and the goals of school programs has consequences for the success of students in schools.⁴

Thus, the school can be viewed as a collectivity, with certain inputs from the community, with various school input characteristics and with outputs in the form of the goals discussed above, namely, cognitive and affective outcomes and physical development. Figure 1 represents a schematic of the model outlined above. This schematic represents a revision of the schematic described by Armor. The solid arrows indicate assumed causal directions. The dotted arrows indicate weaker, but plausible causal directions. In this instance, it is felt that causal directions may be two-way. As an example, the quality of a school may be the factor determining what types of families are

⁴Armor, p. 173.

moving into or out of a community and thus influencing the nature of the community.

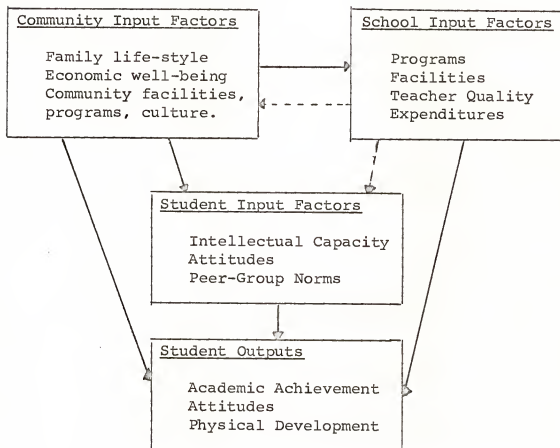


FIGURE 1. A MODEL OF THE EDUCATIONAL PROCESS

Hence, the model states that the outputs of an individual (i^{th}) student at time t are a function (g) of community inputs cumulative to time t ; of the quality and quantity of school inputs consumed by him cumulative to time t ; of his characteristics (initial endowment and attitudes); and of the characteristics of his peers.

Thus $A_{it} = g(C_i(t), S_i(t), P_i(t), I_i)$

where A_{it} = vector of educational outcomes of the i^{th} student at time t .

$C_i(t)$ = vector of community inputs relevant to the i^{th} student cumulative to time t .

$S_i(t)$ = vector of school inputs relevant to the i^{th} student cumulative to t .

$P_i(t)$ = vector of peer influences cumulative to t .

I_i = vector of initial endowments of the i^{th} individual.

The above equation employs mathematical symbolism and can be called a mathematical model. The use of such symbols allows the expression of the conceptual model in a precise fashion and lessens the possibility of ambiguous interpretations.

If the vector A consists of two or more components such as $A = (A_1, A_2, A_3)$ where A_1 = reading achievement, A_2 = citizenship attitudes, and A_3 = social consciousness, then the model generates a system of equations that are to be solved simultaneously. Alternatively, one can conceive of A as a composite of educational outcomes; that is, A is a single number representing an index of educational output. It is also possible for A to represent a single output such as reading achievement. The last two instances describe what has been termed an educational production function. The production function attempts to reveal the relationships

between outcomes of education and the human and material resources which education consumes. Community inputs, peer influences, and initial individual endowments serve as controls in the production function since these factors are necessarily confounded with school inputs. These factors are explicitly brought into the production function, since failure to do so would obscure the true relationships of school inputs to output factors.

The production function allows one to analyze the effects of changes in inputs to the educational process. To properly allocate school resources requires one to discover which resources affect outputs and to estimate the effects that changes in their allocation might have. Policy-makers want information about the possible effect of hiring better trained or smarter teachers, about the influence of small pupil/teacher ratios, and so forth. In short, they want to know what will happen if they change the status quo. Thus, it would seem that in setting school policy and in long-range educational planning, knowledge of the educational production function is essential to efficient resource allocation.

Production function estimation requires a knowledge of the different factors important in producing the product and the interactions between the inputs. A typology of

variables associated with educational attainment involves the following categories: (1) student inputs, (2) community inputs, and (3) school inputs. The first two categories represent non-manipulable inputs into the school system. School inputs can be further sub-divided into: (1) teacher variables, (2) administration variables, (3) facilities, (4) expenditures and (5) types of programs. To the extent that variables representing important determinants of achievement or their proxies are not included in the model, the adequacy of the model is diminished. The fact that a theory of learning is lacking on the educational scene increases the probability of important aspects being left out of the model. But, the key lies in the ability of the model to predict performance even when proxy variables are used. There is ample evidence that the discovery of predictive factors may be useful on practical grounds even if these factors have no causal significance. The appendix lists variables deemed desirable for production function estimation.

Dimensionality of output is a major concern. Previous input-output analyses of the educational process have generally employed some measure of academic achievement as the dependent variable. As Mosteller and Moynihan stated, "Lest it seem that academic achievement must be the

only job of the schools, let us remember that studies do not find adult social achievement well predicted by academic achievement."⁵ Thus it would seem that other educational outputs as suggested in the conceptual model must be brought into the analysis if results are to be meaningful. This can be done in one of two ways. First, the various outputs can be entered separately into the analysis. This leads to a system of simultaneous equations as previously explained. Secondly, the estimation of the educational production process can be based upon some measure of total educational output. That is, in some way the outputs should be weighed by a common factor (utility, priority, social value) in order to obtain a total index of output.

Another issue centers on whether the measures of output employed should represent current levels of performance or represent gains made over a specified period of time. Ideally, it is desired to know what students have gained during the time they have been under instruction, how much of the gain may be reasonably attributed to the instruction, and how much to factors beyond the reach of the school. Attempts at using this approach have fallen short of success due to measurement problems.

⁵Frederick Mosteller and Daniel P. Moynihan, "A Path-breaking Report," On Equality of Educational Opportunity, p. 7.

Constraints and the Educational Process

Any attempt to describe the process of education by production function estimation must recognize various forces (i.e., social, political, economic, legal) that act as constraints upon the process. This has been one of the shortcomings of previous input-output analyses of educational systems and may explain in some degree their lack of impact upon policy decisions.

The most obvious constraint is the budgetary one. Resources for education, as for all other purposes, public or private, are limited. Decision-makers must operate within the framework of a limited budget. Thus, the key policy questions they face are these: when a school changes its inputs in various ways, how much improvement in achievement scores (or other desired good) will be obtained and for how much money? Evaluation of alternative policies necessarily involves consideration of both the effects on output of different changes in the inputs to the production process and the costs of these changes.

It is also true that existing state laws, accreditation requirements, and market demands may serve to inhibit alternative choices among the use of school resources. Examples follow:

1. Pupil-teacher ratio--analysis may show this to have negligible effect, but accreditation requirements prevail.

2. Teachers' salaries--again the effect may be small relative to other variables, but disproportionate increases may be needed to attract and retain qualified teachers due to market demands.

3. Length of school year--analysis may suggest an optimal length different from that prescribed by state law. No doubt, the process of estimating an educational production function might suggest changes in state laws and regulations with respect to education. But more importantly, by considering present constraints, educational decision-makers can affect educational outcomes now, as best they can, until laws and regulations or conditions are changed.

Technical and Allocative Efficiency

As attempts are made to allocate resources to best effect educational outcomes, it is apparent that two types of efficiency are involved. Levin has termed them technical and allocative efficiency. Technical efficiency refers to using the physical resources of a school in such a way that more educational output is achieved than might be done under some alternative utilization. Allocative efficiency refers to using the dollar budget of the school in such a way that more output is achieved than could be attained if a different

combination of physical resources were purchased even when technical efficiency is satisfied.⁶

It is at this point that a major distinction occurs between the concept of an educational production function and an industrial production function. The methods that have previously been used to estimate educational production functions employ an average concept; that is, the final function represents what the average effects are for various inputs. On the other hand, an industrial production function is conceptually designed to express the maximum product obtainable from the input combination at the existing state of technical knowledge. Thus the industrial production function represents a frontier of potential attainment for given input combinations. It establishes the most technically efficient combinations of current practices. On the other hand, if it is desired only to estimate how much output on the average could be obtained for a firm in the industry with a certain set of inputs, then the average concept would obviously be the correct one to employ. But, this function cannot answer questions regarding technical efficiency. It can resolve the question of allocative efficiency at the existing state of technical knowledge.

⁶Henry M. Levin, "The Effect of Different Levels of Expenditure on Educational Output," p. 182.

The Need for Optimization

A number of input-output analyses have been performed in education and these studies have provided a necessary description of the educational system. However, the policy implications of such studies have been hard to realize since they generally have been devoid of cost considerations. A simple example follows to indicate the necessity for cost analysis in conjunction with the production function. Suppose two variables x_1 , x_2 are found to have the following relationship to y :

$$y = 4x_1 + x_2.$$

It appears that x_1 has a greater effect on y than x_2 . Hence, researchers viewing this equation would stress the importance of x_1 in their conclusions. But, suppose further it is known that the prices of x_1 and x_2 are respectively $P_1 = 5$ and $P_2 = 1$. Then it becomes clear that within a limited budget, more can be accomplished by using x_2 rather than x_1 . Suppose that the budget equation happens to be:

$$5x_1 + x_2 = 20.$$

This would allow us to use at most 4 units of x_1 when $x_2 = 0$ or 20 units of x_2 when $x_1 = 0$. The maximum value of y would be $y = 20$ when $x_1 = 0$ and $x_2 = 20$. It is realized that this is a rather naive example, for which the solution is intuitively obvious. However, in a real-life situation,

if one had a production function consisting of 20 or more variables, each having different prices, then it becomes intuitively impossible to know what the best combination of inputs would be. The point is that input-output analyses of schools and educational policy miss the mark if they are done irrespective of price considerations for the various school inputs into the educational process. Conclusions drawn from an examination of a production function may be severely revised as a result of the joint consideration of budget constraints. What is needed, then, is a systematic procedure for determining the best combination of inputs when dealing with large numbers of variables.

Conceptual Considerations in Retrospect

In review, the process of education has been viewed in an input-output framework. It was found that in addition to school inputs, community and student inputs also affect the magnitude of educational outputs. Also, recognition must be made of various constraints that are operating at each level of the process. The most pervasive of these is the budgetary constraint. Finally, given the conceptual framework of the educational process, the task of decision-makers is to optimize educational outcomes subject to various constraints which explicitly and implicitly operate on the system.

Mathematical Considerations

In the recent past there has occurred an emerging interest in applying to education the methods of economic analysis. These methods have been variously called operations analysis, systems analysis, input-output analysis, cost-effectiveness analysis, and cost-benefits analysis. They are viewed as a potential means of increasing the efficiency of educational planning and decision-making. These types of analyses can be utilized in education to deal with such questions as how much of an increase in reading achievement of sixth-grade children is likely to result from any given reduction in the size of sixth-grade classes, all other things being equal. Essentially, these types of analyses allow the planner to view an educational system as a set of input-output or production relationships which can be controlled in a way that will optimize the use of scarce educational resources. The operational form of such analyses usually involves a mathematical model which describes in a quantitative sense the structural characteristics of the phenomena under investigation. In developing a mathematical model, in the present case, it is necessary to consider (1) ordinary least squares versus two-stage least squares, (2) single versus multiple outputs, (3) technical and allocative efficiency, (4) prices of the

school inputs, (5) statistical difficulties in using multiple regression, and (6) strategies for optimization.

Ordinary Least Squares Versus Two-Stage Least Squares

Previously, the educational process was viewed as composed of multiple outputs. This was reflected in the equation

$$A_{it} = g(C_i(t), S_i(t), P_i(t), I_i)$$

where A_{it} is a vector of educational outcomes. Two ways have been identified for working with multiple outputs. One leads to a system of equations (production functions) to be solved simultaneously. The other consists of constructing an index of total educational output so that the educational process may be represented in a single equation. A major difficulty with the latter approach is that some outputs are likely to be causally related to other outputs. Thus, an increase in positive attitudes is likely to lead to an increase in academic achievement which in turn will cause better attitudes and so on. The use of ordinary least squares to estimate the parameters of a single equation with this type of feedback existing among the outputs will probably lead to biased estimates. When the above situation exists a better approach is to employ

simultaneous equations. The parameters on these equations can be estimated using two-stage least squares.⁷

Single Versus Multiple Outputs

If one adopts the simultaneous equations approach, then each equation estimates a single output. Difficulties arise when one attempts to allocate resources so as to maximize all outputs. Most likely, not all outputs can be weighted equally; that is, one may not be willing to assign the same value or utility to each of the outputs. Thus, one is required to subjectively determine the weights that describe the utility of each output. Hence one seeks to maximize a utility function of the various outputs. It is at this point that trouble is likely to occur. Gilbert and Mosteller stated, ". . . setting many desirable maxima makes it hard to know what programs might possibly lead to achievement of these goals. A well-known mathematical theorem states that one cannot ordinarily maximize two or more variables simultaneously."⁸ Consequently, it would

⁷ Henry M. Levin, "A New Model of School Effectiveness," in Do Teachers Make a Difference? (Washington, D.C.: U.S. Government Printing Office, 1970).

⁸ John P. Gilbert and Frederick Mosteller, "The Urgent Need for Experimentation," On Equality of Educational Opportunity, p. 382.

be vain to hope to maximize the outcomes of each student in several different areas. In the utility function, only a few of the outputs would be maximized, at the expense of other outputs.

The above seems to suggest that analysis might be more fruitful if it were restricted to a single output, which is not a composite of other outputs. That this should be so, seems to follow for additional reasons. McNamara stated,

Mathematical applications to management should stay clear of large general models and concentrate on specific problem areas. The mistake that educators have made for years involves trying to solve huge problems that do not have immediate or feasible solutions. If the general area of operations analysis is to persevere in educational management, then the appropriate approach seems to be the consistent application of these techniques to small problems.⁹

Such an approach recognizes that resources should be allocated to programs. As an example, given a reading program, interest centers on how resources might be allocated to maximize reading achievement.

No doubt, if the focus is on reading achievement only, analysis is likely to indicate a distribution of resources that maximizes reading achievement at the expense of other

⁹James F. McNamara, "Mathematical Programming Models in Educational Planning," Review of Educational Research, XLI (December, 1971), 440.

outputs. This need not be, however. As the various constraints operating on the reading program are considered, constraints can also be introduced that establish minimum standards for other outputs. For example, in the reading program, analysis may suggest that the ratio of students to counselors be increased so that more resources are available for increasing the training of classroom teachers. Such a decision would likely have a negative influence on certain affective outcomes. Thus, in the analysis, a constraint can be introduced that maintains a minimum standard for the student-counselor ratio. This standard would be determined either as a subjective judgment or as a result of previous analyses in which the appropriate affective outcomes were the outputs under consideration.

Technical and Allocative Efficiency

In deciding upon the methodology to employ in estimating the production function distinctions must be made between technical efficiency and allocative efficiency. It will be recalled that the industrial production function expresses the maximum product obtainable from the input combination at the existing state of technical knowledge. The process of using ordinary least squares to estimate the production function, on the other hand, expresses the

average product obtainable from a firm in the industry. Educational production functions have generally been estimated using ordinary least squares analysis. The reason for this is that given the lack of a comprehensive theory of learning, it is not known if all the relevant inputs have been identified. Likewise, it has not been possible to provide adequate data for some of the inputs presently identified.

Given this state of affairs, contentment must rest with the way things are rather than the way things should be. In this context, the average concept is the correct one to apply since it can answer questions of allocative efficiency at the existing state of technical knowledge. For purposes of optimization, then, ordinary least squares should be used for estimating the production function. If the output being considered is perceived to have feedback effect on certain independent variables in the equation, then a system of simultaneous equations must be solved in which affected independent variables become the dependent variables in other equations. This system is solved using two-stage least squares. The process of using ordinary least squares or two-stage least squares to estimate the parameters of one or more equations is more generally termed multiple regression analysis.

It should be mentioned, at this point, that multiple regression analysis can provide clues regarding the technical efficiency of the units observed. Each unit in the analysis is composed of a particular combination of input values and an output value. Multiple regression analysis estimates what the average output is with this combination of input factors. Figure 2 illustrates this process. Estimated output is plotted against the actual output observed. If the units of data represent the mean scores of schools on some educational outcome, then it can be argued that in a group of schools working with equivalent inputs (community, student, school), these being indicated by the estimated output, every school in the group should be able to do what the most productive schools in that group have done.

This allows questions of technical efficiency to be answered. By observing the high productive and low productive schools in a given group, a whole series of questions can be asked about the characteristics of high productive schools that make them different from the low productive schools--questions about school organization, courses offered, methods of instruction, teacher attitudes and behavior, special services, community involvement, quality of facilities, level of financial support, and the

like. Results of such analyses would help to identify new factors that are related to output. Incorporation of these new factors into the production function advances the level of technical knowledge.

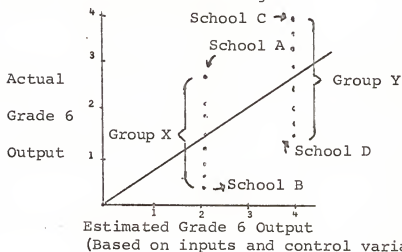


FIGURE 2. METHOD FOR INVESTIGATING TECHNICAL EFFICIENCY

Determining Prices of the School Inputs

Thus far, it has been suggested that multiple regression analysis be employed for estimating the production function and that consideration be given to maximizing single outputs only. Also, as was seen earlier, one of the constraints to be considered is the budgetary constraint. Having identified the relevant school inputs in the production function, their prices must be estimated. In linear form, the budget constraint can be expressed as

$$B = \sum p_i x_i \quad (i = 1, 2, \dots, n)$$

where B represents the total budget

x_i = i^{th} school input in the production function

P_i = price of the i^{th} input per unit.

The costs of some variables are obvious and relatively easy to obtain, such as teachers' salaries and instructional expenditures per unit. For other variables, the costs appear to be intractable using traditional accounting methods. However, Cohn, Katzman, Levin and Riew,¹⁰ by using least squares techniques, were able to obtain some idea of how changing a unit of a non-priced variable can affect total cost. Cohn determined the relationship to total per school costs on such variables as

Z_1 - average number of college semester hours per teaching assignment,

Z_2 - average number of different subject matter assignments per high school teacher,

Z_3 - average class size,

Z_4 - number of units of subjects offered.

¹⁰Elchanan Cohn, "Economies of Scale in Iowa High School Operation," Journal of Human Resources, III (Fall, 1968); Martin T. Katzman, "Distribution and Production in a Big City Elementary School System," Yale Economic Essays, VIII (Spring, 1968); Henry M. Levin, "A Cost-Effectiveness Analysis of Teacher Selection," Journal of Human Resources, V (Winter, 1970); and John Riew, "Economies of Scale in High School Operation," Review of Economics and Statistics, XLVIII (August, 1966).

Thus it appears that costs of the various school variables can be determined with the aid of least squares analysis.

Statistical Considerations in using Multiple Regression

Both the production function and the budget equation are to be estimated using multiple regression techniques. It is necessary, then, that a critical discussion of all the statistical issues relevant to multiple regression analysis be considered. Thus far, the terms multiple regression and least squares analysis have been used. The problem of predicting or estimating the value of a dependent variable given various independent variables is correctly termed multiple regression. Ordinary least squares is one technique for determining the regression equation. Since ordinary least squares will generally be employed in the estimation process, the two terms will be used interchangeably, unless otherwise noted.

Ideally, the process of estimating a regression equation should be based on a mathematical model including all of the factors believed to have an influence on the dependent variable. In the linear case, a typical regression equation has the form:

$$y = b_0 + b_1x_1 + b_2x_2 + \dots + b_nx_n + \epsilon$$

where $\hat{y}$ = estimated y (dependent variable)

x_i = independent variable

b_i = regression coefficient ($i = 1, 2, \dots, n$)

b_0 = constant term (for scaling purposes)

ϵ = error term or residual ($\epsilon = \hat{y} - y$).

It is important that sufficient variation be observed in all of the independent variables that valid estimates of the regression coefficients may be made. Otherwise, existing differences among schools may not be great enough to reveal the importance of school resources to a given output. Technically, one is not allowed to extrapolate the results of a regression analysis beyond the range of observed data. At the same time, the effects of certain resources may not be apparent in the range of existing variation. Reducing the pupil/teacher ratio, for example, may make no difference until instruction can be really individualized, which might require a pupil/teacher ratio of less than 10:1. Also, suppose that output is functionally related to both community and school inputs. Further, suppose that school inputs vary only slightly in a particular sample and that home inputs vary widely. Statistically, most of the variance in output would be attributable to community factors. Thus, it becomes imperative that in any sample used to estimate the production function, the population extremes be well represented.

Interpretation of regression coefficients

The regression coefficient has the following interpretation: b_1 represents the amount of change in y that can be associated with a unit change in x_1 , with the remaining independent variables held constant. Since in a multiple regression analysis the remaining variables are controlled, b_1 is termed a partial regression coefficient. An interesting property of the partial regression coefficient is that regardless of whether x_1 is influenced by or influences other independent variables in the equation, the same regression coefficient for x_1 will be obtained.¹¹ The size of the regression coefficient depends on the unique contribution of the independent variable to the prediction of the dependent variable plus an apportioned amount of the influence it shares with other variables in and out of the equation.

Multiple regression analysis enables one to find the relationship between a given input factor and an output, with all other factors controlled. More precisely, this means that the partial regression coefficient is a weighted average of regression coefficients obtained by fixing

¹¹Robert L. Linn and Charles E. Werts, "Assumptions in Making Causal Inferences from Part Correlation, Partial Correlations, and Partial Regression Coefficients," Psychological Bulletin, LXXII (1970), 309.

control variables at all possible values in the range observed and computing separate regression coefficients. Any indirect effects that the control variables may be having on y acting through x_i are isolated. Thus, it is important that variables entering the analysis are distinct, that is, they are not measuring the same factor and hence are not identical. Otherwise, controlling for them becomes tantamount to partialling the relationship out of itself. This is known as the partialling fallacy.¹² The use of stepwise regression avoids the inclusion of variables that are not distinct. Controls on distinct independent variables which are unnecessary will not produce misleading results as long as we do not attempt to control on a dependent variable. The size of a regression coefficient may still be influenced by variables that have not explicitly been brought into the analysis. This is discussed in more detail in the section on causality and multiple regression.

Regression models and goodness of fit

By correlating the estimated y 's with the actual y 's, a measure of the goodness of fit of the multiple regression

¹²Robert A. Gordon, "Issues in Multiple Regression," American Journal of Sociology, LXXIII (March, 1968), 592.

equation to the data is obtained. Such a correlation is termed multiple correlation $(R_{Y.X_1 X_2 \dots X_n})^2$. $R_{Y.X_1 \dots X_n}^2$ gives the proportion of variability in the dependent variable explained by the regression relationship. If A is a set of original variables and B is a set of added variables, any increment to an $R_{Y.A}^2$ due to the addition of X_B can be tested for significance by the F ratio

$$F = \frac{(R_{Y.A,B}^2 - R_{Y.A}^2) / b}{(1 - R_{Y.A,B}^2) / (n - a - b - 1)}$$

df = b and (n - a - b - 1).

$R_{Y.A,B}^2$ is the incremental R^2 based on a + b independent variables where

a = number of original variables

b = number of added variables.

When R is based on sample data, there is a tendency for it to be systematically biased upwards. This happens because the least squares process capitalizes on any sampling error that overestimates zero-order correlation coefficients. This is known as the problem of shrinkage--the tendency for R to decrease as the sample grows larger. A correction formula is given by

$$\overline{R^2} = 1 - (1 - R^2) \left(\frac{N - 1}{N - k} \right)$$

$\overline{R}$ = unbiased estimator of population R

R = multiple correlation found in a sample of size N

k = number of independent variables.

Of course, if one is dealing with population data, shrinkage is not a problem.

Multicollinearity

Past studies that have estimated educational production functions have generally exhibited evidence of high degrees of interrelationships among the independent variables. This is known as the problem of multicollinearity. This affects the regression analysis by increasing the standard errors of the regression coefficients for the collinear variables; that is, the estimated regression coefficients exhibit more variance and in a sense are unstable. Small sampling variations among the correlations of a highly related set of independent variables can create large variations among their regression coefficients. Still, the coefficient values themselves are not biased. Attempts to avoid the effects of multicollinearity have involved one or more of the following: (1) introducing extraneous information, (2) stratifying, and (3) providing for simultaneous relationships. Another alternative would be to study an

entire population rather than a sample. As an example, if one were concerned with educational policy for a given state, it would be almost as easy to conduct an analysis of data for all the districts in the state, assuming that appropriate data collection techniques were carried out by the state department of education. Since in a sample regression equation the regression coefficients are unbiased estimators, it would follow that in the population regression equation the regression coefficients would represent the true effects for the variables present.

The need for nonlinear models

Most applications of multiple regression employ the linear additive model. However, for purposes of estimating production functions, the following limitations of such a model are apparent.

1. The model does not allow for economies of scale.
2. It does not allow for interactions between inputs.
3. It implies that the marginal effect of a given input is the same regardless of the level of usage.

What the above suggests is that in estimating educational production functions, most likely nonlinearities in the observed data must be considered. The provision of any given power u of x_1 , that is x_1^u , allows for $u-1$ bends in the

regression curve of y on x_1 . In most research in the behavioral sciences, provision for more than one or two bends will rarely be necessary. A simple parabola may often give a reasonably good fit to the data, especially when it is realized that the curve may be quite flat and the data need not extend far enough to complete the bend (i.e., the parabola may be a reasonable fit within the limits of variation given in the problem). To obtain a measure of the goodness of fit to the parabola, the multiple R between y and x_1 , x_1^2 is used. The difference between R^2 and $r_{yx_1}^2$ (assuming linearity) will give a measure of the degree to which ability to predict has been improved.

Another type of nonlinearity is often encountered in research studies, namely, the concept of interaction. The importance of this concept is revealed by Mosteller and Moynihan in their reanalysis of the Coleman study (EEOR). It is recalled that Coleman concluded that the relative effects of school resources are small compared to community and student inputs. Mosteller and Moynihan stated:

To the simple of mind or heart, such findings might be interpreted to mean that 'schools don't make any difference'. This is absurd. Schools make a very great difference to children. Children don't think up algebra on their own. . . . But given that schools have reached their present levels of quality, the observed variation in schools was reported by

EEOR to have little effect upon school achievement. This actually means [that of] a large joint effect owing to both schools and home background (including region, degree of urbanization, SES, ethnic group) little is unique to school, or homes. They vary together.¹³

Cohen stated that this joint effect is carried by a variable z of the form $z = X_i X_j$.¹⁴ This is known as the first order interaction effect. Interaction terms that measure the joint contribution to explained variance of two or more variables can be very important. If two variables are highly intercorrelated, the first entered into the regression analysis will be assigned both its unique contribution to the explained variance and its jointly explained variance with all other variables (the interaction terms). This is precisely what happened in the Coleman report.

If one were to employ an interaction term in a simple three-variable model the equation would take the following form:

$$x_3 = a_{13}x_1 + a_{23}x_2 + a_{123}x_1x_2.$$

How does one interpret the coefficient a_{123} ? The partial derivative of x_3 with respect to x_1 expresses the slope of the regression surface with other independent variables held constant (i.e., amount of change in x_3 associated

¹³Mosteller and Moynihan, p. 21.

¹⁴Jacob Cohen, "Multiple Regression as a General Data-Analytic System," Psychological Bulletin, LXX (1968), 436.

with a unit change in x_i). Thus, in the above equation:

$$\frac{\partial x_3}{\partial x_1} = a_{13} + a_{123}x_2$$

and the amount of change expected in x_3 is dependent upon the level of x_2 . In an educational production function this could mean that the effect on performance of increasing the percentage of students from white-collar homes is not independent of the percentage of teachers in a school who are experienced and who have a master's degree. Consider

$$\frac{\partial}{\partial x_2} \left(\frac{\partial x_3}{\partial x_1} \right) = a_{123}, \text{ this expresses the rate of change}$$

of $\frac{\partial x_3}{\partial x_1}$ when x_2 changes. In other words, this is the effect of x_2 on the effect of x_1 on x_3 .

Thus the concept of interaction consists essentially of a residual category of all types of effects that are non-additive. It always refers to the joint effects of two or more independent variables on some particular dependent variable. Since nonadditive relationships constitute a residual category, it would be rather surprising if interaction effects could always be interpreted in a simple manner. Nevertheless, an interaction model would enable us to determine whether variables are complements (that is, act together to improve performance) or substitutes (namely,

one variable has a greater effect in the absence of some other variable than it has in its presence). The implications for educational decision-making is that it would allow the use of input-background interaction in conjunction with the demographic composition of a student body to achieve desired performance levels.

Reliability of measurement

In conducting multiple regression analyses one must always be concerned with the reliability of measuring instruments. If an output is unreliably measured and if the errors are random, the increase in the standard deviation of output offsets the decrease in the correlation between output and input, leaving the regression coefficient unbiased. Given

$$y = (r_{yx}) \frac{(S_y)}{(S_x)} (x) + \epsilon ,$$

the above is apparent. Though the regression coefficient is unbiased, an increase in the standard error of estimate and a corresponding decrease in the multiple R can be expected. There are formulas that allow one to correct the attenuation in correlations. If, on the other hand, random errors are made in measuring x , r_{yx} will be underestimated and S_x will be overestimated, leading to biased results. This type

of problem can also be corrected but the work involved is tedious.¹⁵

Quite frequently the level of data treatment will call for aggregated data. If the unit of analysis is at the district level, then aggregated or mean measures on the variables would be observed. Shaycroft has given a formula for estimating the reliability of group means.

$$r_{\bar{a}\bar{a}} = 1 - \frac{1 - r_{aa}}{n} \cdot \frac{S_a^2}{S_{\bar{a}}^2}$$

where $r_{\bar{a}\bar{a}}$ = reliability of group means

r_{aa} = reliability of individual responses

S_a^2 = variance of individual responses

$S_{\bar{a}}^2$ = variance of group means

n = number of individuals in a group.¹⁶

Suppose $r_{aa} = .50$ (reliability of some psychological tests),

$S_a^2 : S_{\bar{a}}^2 = 10 : 1$, and $n = 100$; then $r_{\bar{a}\bar{a}} = .95$.

¹⁵Albert Mandansky, "The Fitting of Straight Lines when Both Variables are Subject to Error," Journal of the American Statistical Association, LIV (1959), 173-205.

¹⁶Marion Shaycroft, "The Statistical Characteristics of School Means," Studies of the American High School, ed. by John C. Flanagan et al., Cooperative Research Project 176, Project Talent, University of Pittsburgh, 1962.

Actually, for most purposes, it can be seen that when n is sufficiently large (100 or more), r_{aa} is close to 1. Consequently, when dealing with aggregated data, reliability is not seen to be a problem.

Use of gain scores

Many authorities have expressed a preference for measures of output that are "value added." If Y is a measure of output, it would be preferable to consider Y as a measure of the net gain achieved within a specified period of time. However, the reliabilities associated with gain scores are always less than that of either of the scores used to obtain a gain score. McAfee studied the reliabilities of gain scores using Monte Carlo simulation.¹⁷ It was found that the average reliability of gain scores with a pre-test reliability of 0.90 is only 0.16. Thus, at the level of individual data treatment, the use of gain scores cannot be justified. Once again, however, if aggregated data are used, the problem of reliability is nearly resolved. Using the above example of Shaycroft's formula and replacing r_{aa} with $r_{aa} = 0.16$ gives $r_{aa} = 0.92$.

¹⁷Jackson K. McAfee, "Problems in Measuring Change," Unpublished paper, University of Florida, 1972.

There remains one other difficulty with the use of gain scores. In using gain models, the school effects are probably understated, since prior performance levels which are due both to school and environment are summarily subtracted out. Psychologists note that changes in a child's self concept are usually the result of similar experiences repeated over a long period of time. Thus, increases in a child's achievement in a given year may have been the result of prior school influences operating over a period of years. It may be best to deal with levels of outcome only as an expression of the cumulative influences of schooling to that point in time.

Population data vs. sample data

The possibility of using either population or sample data has been considered. The use of sample data is often done with the goal of generalizing results to the population sampled. The use of inferential statistics in multiple regression requires a set of restrictive assumptions.

1. The variance must be homogeneous, that is, the variance of y values about the regression surface must be the same at all combinations of x values.

2. The errors ($\epsilon = \hat{y} - y$) must be independent of each other.

3. The x values must be measured with essentially no error.

4. The distribution of y values must be normal. The use of population data requires only that assumption (3) be satisfied.

Causality and multiple regression

In assessing the results of a multiple regression analysis, distinctions must be made between associated effects and causal effects of the independent variables on the dependent variable. It is one thing to say that an increase in y of b units is associated with an increase of one unit in x_i , and another to say that an increase of b units in y is caused by an increase of one unit in x_i . To make a statement of causality requires further assumptions regarding the specification of variables to be included in the model.

In particular, if the regression model is used for simulation purposes, the following must hold true if regression coefficients are to be interpreted causally.

1. All variables which might affect the dependent variables are either included in the regression equation or are uncorrelated with the variables which are included. The absence of relevant influences will, in general, lead

to 'specification error' (i.e., bias in the calculated regression coefficients due to incorrect specification of the structural model). This also implies that the regression residuals (ϵ) are uncorrelated with any of the x_i . The error term represents all those unmeasured implicit factors which influence output, but are assumed or known to be uncorrelated with any of the x_i in question.

2. All x_i are measured without error. Random measurement error in output should not bias the regression coefficients.

3. Terms are included in the regression to handle any curvilinear or interactive effects.

4. The dependent variable has no effect on any of the independent variables. When this happens, one must solve a system of simultaneous equations to obtain unbiased estimates of the regression coefficients. One must also use two-stage least squares instead of ordinary least squares in the estimation process.

Given that most estimates of educational production functions have been based on cross-sectional data in a survey-type analysis, it would be improper to assert causality to the regression coefficients since their sizes are so sensitive to the proper specification of the equations. If relevant influences have not been brought explicitly into

the equation, this can lead to overestimation of the relationships between output and input variables in the equation. Also, the independent variables used may be proxies for the causative variables. Thus, an increase in teacher salary may not be accompanied by a corresponding increase in all the other attributes for which teacher salary is serving as a proxy. It can be said that the regression coefficients are unique regardless of whatever causal interpretation is given to the correlations between independent variables that are in the equation. Suppose two independent variables are correlated. Causally, this correlation might correspond to a direct effect of x_1 on x_2 or of x_2 on x_1 , or to an indirect effect of x_1 on x_2 through x_3 , or to a simultaneous effect of x_3 on both x_1 and x_2 . But the regression coefficient b_1 , the effect of x_1 on y , will be the same whether x_1 causes x_2 or x_2 causes x_1 .

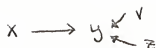
Where causal inferences are desired, one should select variables that are distinct and avoid the use of indices that are thought to be measures of the "same" underlying variable. Generally, factor analysis has been employed to resolve redundancies among the indicators of a given factor. Blalock stated that factor analysis techniques and the interpretations given to the factors extracted ordinarily presuppose certain limited kinds of causal

models.¹⁸ In particular, it is assumed that there are no direct causal links among the indicator variables and that these measured variables are caused by the underlying variables, making the intercorrelations among the indicators completely spurious. Factor analysis may therefore not be appropriate for census data or other types of analyses in which some of the measured variables are caused by other measured variables. But given the intransigence of the regression coefficient to causal directions among measured variables, stepwise regression can prevent the inclusion of redundant information and thus selects the indicator that is optimally related to the dependent variable.

Experimental design vs. survey design

It is doubtful that in estimating a production function all of the relevant influences on output can be identified or, if identified, can be properly measured. In other words, it is doubtful that all the relevant variables can be controlled. Another alternative over rigid controls lies in the experimental tradition of randomization. Randomization does not control for implicit variables. Rather, it transforms the causal scheme into the form

¹⁸Hubert M. Blalock, Causal Inferences in Non-Experimental Research (Chapel Hill: University of North Carolina Press, 1961), p. 168.



where v and z are the implicit influences. They continue to operate on y but do not systematically affect the estimation of the relationship between x and y as measured by the regression coefficient. They do increase sampling error.

According to Blalock, of the implicit factors that are confounded with variables x_i in their influence on y , there exist two types:

1. Forcings--those variables that are impinging from the outside environment;
2. Properties--those variables that are conceived to be properties of the system at the time of observation.¹⁹

Randomization reduces the effects of property variables; however, it does not handle forcing variables. This parallels the Campbell-Stanley tradition regarding sources of external invalidity. In school effects studies the interactions of student selection and school variables, testing and school variables, and various reactive arrangements cannot be handled with randomization. Thus

¹⁹ Ibid., p. 22.

randomization allows simplifying assumptions to be made about a large class of 'property variables' that can be made to operate independently of the causal variables under study. But other 'forcing variables' cannot so readily be ruled out. Even in experimental designs, simplifying assumptions must always be made if causal models are to be evaluated. In short, causal laws can never actually be demonstrated empirically. This is true even where experimentation is possible.

Thus, even though the process of experimentation is viewed with limitation, it is certainly a far more effective device for eliminating alternative plausible causal schemes than is the survey study. However, in the carrying out of large-scale studies for the estimation of educational production functions, recourse to experimentation is not likely to be feasible. This is the current state of affairs in educational research. If there is any consolation, it lies in a statement by Gilbert and Mosteller:

Our own experiences with observational studies . . . has been that the controlled study usually does not contradict the observational study, but it does clarify matters and make them firmer.²⁰

²⁰ Gilbert and Mosteller, p. 373.

Strategies for Optimization

Given that an educational production function has been estimated using multiple regression analysis, the regression coefficient assesses the effect of an independent variable on output with the proviso that other variables in the equation remain unchanged. Unfortunately, the condition that 'other things remain unchanged' cannot be met in the real world. A policy of raising teachers' salaries would probably change the monies spent on other educational resources--resources that might be necessary to the success of the student in the school. What is needed is a process that assesses the effects of variables acting in all their simultaneity, a process that specifically notes the consequences on other resources when a given resource is changed.

Essentially, interest centers on a situation in which the use and level of resources are constrained by various factors. In particular, the budget equation serves as a primary constraint in the educational process. If for a given budget, all dollars are already charted for consumption, then an increase in any resource must necessarily mean decreases in others.

At the same time, subject to various constraints, decision-makers are attempting to optimize the allocation of resources so as to maximize educational outcomes.

Optimization is the process of finding a best solution among several feasible alternatives. What is needed, then, is a systematic procedure for determining the best combination of inputs subject to certain constraints. Such a procedure is mathematical programming. If a model of the educational process describes a linear system such as

$$y = \sum b_i x_i$$

subject to the following constraints

$$\sum a_{ij} x_i \leq c_j$$

then the techniques of linear programming can determine optimal solutions. The x_i are the variables that can be manipulated to achieve the desired objective. $y = \sum b_i x_i$ is called the objective function, since the objective is to optimize y . Most frequently, non-negativity restrictions will be placed on the x_i (i.e., $x_i \geq 0$ for all i). The constants (a_{ij} , b_i , c_j) are termed parameters; they are factors that effect the objective function but cannot be manipulated as are the variables. Constraints not only provide regions of acceptable values of the variables but also provide a mechanism for relating the mathematical expression to real world conditions. The important contribution of this approach is that it realistically views the process of education as a system constrained by political, social and economic considerations.

As stated previously, the linear approach simply ignores such phenomena as economies of scale and non-additive interactions among input variables. Actually, the process of education might better be approached using nonlinear programming, problems in which one or more terms of either the objective function or any one of the constraints is represented by a nonlinear function. Generally, the system can be expressed as:

$$y = g(x_1, x_2, \dots, x_n)$$

subject to

$$\sum f_i(x_1, x_2, \dots, x_n) \leq C_i.$$

Thus, programming is the mathematical method for the analysis and computation of optimum decisions which do not violate the limitations imposed by inequality side-conditions.

Associated with each programming problem is a dual problem. The process of finding the optimal values for the original problem also gives the optimal solution for the dual problem. Without discussing all of the ramifications of this concept, there is one important implication for production function analyses. In the process of maximizing the production function (objective function), a level of output at a minimum cost (budget equation) is attained. This is to say that there is no other way of allocating resources to the inputs such that this level of output

can be attained for a lower cost. Given then, that cost has been minimized at an optimal level of output, the budget can be incremented by x units and the effect observed on output.

Up to this point, consideration has been given only to maximizing output given a specific budget. But the process of incrementing the budget has broader implications for policy analysis. Specifically, educational decision-makers should follow this approach to financing educational programs:

1. What are the outcomes that are desired and at what levels?
2. What programs are necessary to achieve the outcomes that are desired?
3. What resources are required to carry out the programs?

Given this approach towards the financing of educational programs and the optimization techniques described above, decision-makers are in a position to estimate the additional resources required to bring about incremental improvements in educational outcomes. As an example, suppose the statewide average on reading achievement for all twelfth-grade students is 11.7 years. Suppose further that a production function has been established between

reading achievement and various inputs into the educational process. Then educational leaders are in a position to determine the minimal cost to bring about any desired level of improvement. If the decision is made, for instance, to increase the reading level to 12.4 years, then an optimal combination of inputs can be determined that will minimize costs. The advantage of this approach is that decision-makers can have some idea of what additional resources allocated to education will buy.

In the process of developing a mathematical model, the idea of integrating the concepts of multivariate statistics and mathematical programming may seem to some to be a novel approach. It is one, however, that was anticipated as early as 1963. At that time Wegner stated:

Multivariate statistics is concerned with the estimation of parameters and the determination of the structure of relations between variables. Mathematical programming assumes a mathematical model of known structure and is concerned with the determination of optimal policies subject to known structural restrictions. Each topic can supplement the other, and the union of the two disciplines would provide a more powerful tool than either discipline could provide in itself . . . problems requiring a combination of the two disciplines are becoming increasingly common.²¹

²¹P. Wegner, "Relationship Between Multivariate Statistics and Mathematical Programming," Applied Statistics, XII (November, 1963), 146-150.

An Algorithm for Solving Nonlinear Programming Problems

In estimating the production function and the budget equation, multiple regression analysis was an appropriate methodology. It is necessary to determine an appropriate methodology to solve a nonlinear programming problem in which the production function is the objective function and the budget equation is one constraint. Other constraints would include the ranges of the variables since predictions cannot be extrapolated beyond the ranges observed. Nonlinear programming problems generally fall into two classes, those in which the constraints are linear and those which are not. Since the budget equation considers questions of economies of scale, there is every likelihood that the equation will include nonlinear terms. The most crucial question regarding nonlinear problems deals with the nature of the objective function. If interest centers on maximizing the objective function, then questions of concavity must be answered. A concave function is defined as a function such that a straight line segment joining any two points on the surface must lie below the surface. Figure 3 illustrates a concave function. Figure 4 illustrates a function which is not concave. The figures also illustrate the basic difference between the two functions. In Figure 4 the

function attains two local maximum values M_1, M_2 . This means that within small neighborhoods of M_1 and M_2 , these points are the maximum values. In this case M_2 is the global maximum since it is the largest value f assumes in the region. In Figure 3, on the other hand, only one local maximum is observed. For a concave function, a local maximum is always a global maximum over a given range.

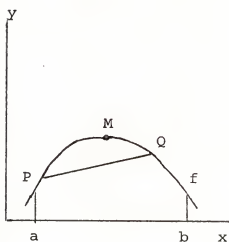


FIGURE 3. AN EXAMPLE OF A
CONCAVE FUNCTION f

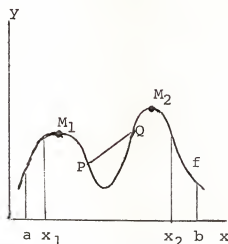


FIGURE 4. AN EXAMPLE OF A
NONCONCAVE FUNCTION f

This illustrates the difficulty of using a nonlinear programming algorithm when the objective function is not concave. The algorithm always converges to a local optimum whether or not it is the global optimum. Thus if the algorithm is started at x_1 the solution converges on M_1 . Starting at x_2 , it will converge on M_2 . When dealing with

a nonconcave function, theoretically one would need to start the algorithm at each point in the range to be assured of attaining the global optimum. Practically, by starting the algorithm at various points in the range the global optimum can be approximately determined correct to a certain number of decimal places. The number of points used determines the accuracy of the estimates. Of course, all of this is avoided if the function is concave, for then any local optimum is a global optimum.

An all-purpose algorithm for solving nonlinear programming problems with nonlinear constraints is the SUMT algorithm. SUMT stands for Sequential Unconstrained Minimization Technique. The technique was perfected by Fiacco and McCormick.²² The general programming problem is to determine a vector $\mathbf{x}$ that solves:

minimize $f(\mathbf{x})$, subject to $g_i(\mathbf{x}) \geq 0$; $i = 1, 2, \dots, m$.
 $\mathbf{x} = (x_1, x_2, \dots, x_n)$.

(Note, a maximization problem can always be transformed to a minimization problem by multiplying through the objective function by a (-1) .)

²²Anthony V. Fiacco and Garth P. McCormick, "Computational Algorithm for the Sequential Unconstrained Minimization Technique for Nonlinear Programming," Management Science, X (July, 1964), 601-617.

This problem is solved by transforming it into a sequence of unconstrained minimization problems.

Define the function

$$P(x, r_1) = f(x) + r_1 \sum \frac{1}{g_i(x)}$$

where r_1 is a positive constant. Proceed from x^0 (starting point) to a point $x(r_1)$ that minimizes $P(x, r_1)$ in the feasible domain (set of points satisfying the constraints of the problem).

Form the new function,

$$P(x, r_2) = f(x) + r_2 \sum \frac{1}{g_i(x)} ; 0 < r_2 < r_1.$$

Starting from $x(r_1)$, determine the minimum of $P(x, r_2)$. Continuing in this manner, a sequence of points $\{x(r_k)\}$, $k = 1, 2, 3 \dots$ are generated that respectively minimize $\{P(x, r_k)\}$ and $r_k \rightarrow 0$ as $k \rightarrow \infty$. This sequence of P - minima converges to an optimum of the original problem; that is, $x(r_k) \rightarrow \bar{x}$, where $\bar{x}$ is the optimal vector and $f(x(r_k)) \rightarrow f(\bar{x})$, as $r_k \rightarrow 0$.

The principal technique used to minimize P at step i is to evaluate the gradient of P at $x(r_{i-1})$ and then to descend the gradient a fixed distance. The gradient gives the direction of steepest descent of the surface P at a given point and thus suggests the optimal direction to

proceed in order to attain the minimum value of the P function. When the original problem is not concave, it is necessary to start at several points to determine an approximation to the global optimum.

Mathematical Considerations in Retrospect

In review, a detailed discussion of mathematical considerations relative to the development of a mathematical model has taken place. As a result of this discussion, several generalizations can be made.

1. Analysis should be confined to a single output. By employing appropriate constraints, minimum standards to be achieved by other outputs can be set.

2. The use of a nonlinear model that incorporates higher degree and interaction terms should be considered. This is necessary to consider questions of economies of scale and non-additive effects.

3. The use of gain scores at the level of the individual is not justified. Such scores are simply not reliable. Aggregated gain scores are reliable for sufficiently large n , but their use is dismissed on the grounds that it leads to underestimation of the effects of school variables.

4. The measurement of independent variables must essentially be errorless. Random error in measuring the dependent variables does not bias the regression coefficients, but does increase the standard error of estimate.

5. Where possible, one should use population data rather than sample data. The use of sample data requires additional assumptions regarding the shape of the distribution (i.e., normal) and the homogeneity of variance. Furthermore, if independent variables are interrelated, multicollinearity may affect the stability of the regression coefficients. If population data are used, the stability of regression coefficients is affected only by specification error (assuming no measurement error in the independent variables).

6. For analysis to be meaningful it is necessary that the variables be constrained by real world conditions. This implies the use of mathematical programming.

7. The production function should be estimated using cross-sectional or longitudinal data in a survey-type design. While the experimental approach is more effective in eliminating plausible alternative causal schemes, the process of randomization and its concomitant, sampling, may lead to unstable regression coefficients.

8. The experimental approach should be followed in investigating the residuals of production function analysis (i.e., the question of technical efficiency). This leads to the discovery of previously unknown implicit factors that influence output and are correlated with certain inputs. Their inclusion in the regression equation allows us to control for their effects on other inputs, which lessens specification error and improves the ability of the model to predict.

A Mathematical Model

Given a conceptual framework for the process of education, a mathematical model is presented which translates these conceptualizations into statements capable of quantitative analysis. Given the educational production function:

$$A_{it} = g(C_i(t), S_i(t), I_i, P_i(t));$$

for a given set of community, peer-group, and individual inputs, the above can be transformed to

$$A_{it} = g(k, S_i(t))$$

where k is a constant. Thus for a given set of school inputs, the problem is to maximize

$$A_{it} = g(k, S_i(t))$$

subject to

$$\sum f_i(k, s_i^{(t)}) \leq B_i.$$

The production function is estimated by using multiple regression analysis, and the entire system is solved by methods of mathematical programming.

CHAPTER III

EMPIRICAL APPLICATION OF THE MODEL

In light of the prevailing emphasis on modeling the educational process, there is an emerging literature that advocates the use of mathematical models as a means to increasing the efficiency of educational planning and decision-making. Too often, however, the intent is simply to focus on the advantages of applying models rather than to provide empirical research that illustrates the unique contributions of such models in generating solutions for real and immediate educational problems. As a result, some rather sophisticated models have appeared in the literature, the mathematics of which have awed even the most interested observers with less than a college degree in mathematics. Also, the developers of such models have failed to provide means by which parameters evident in the model could be estimated. No matter how close the model may mirror the conceptual framework, any estimation of parameters on other than an objective basis defeats

the primary purpose of model development. Any subjective determination of the values of the parameters only obscures the intended objectivity.

Thus the process of estimating the educational production function necessarily requires a statistical dimension for parameter estimation. At the same time, a statistical dimension necessarily restricts the complexity of model design. Thus the choice which a designer must face is developing highly sophisticated models, for which parameters cannot be objectively determined using current means, or developing less sophisticated models with a statistical dimension.

Considering the urgent need to demonstrate the utility of such models, the choice made is the model presented in the previous chapter. In this chapter, using real data, an empirical illustration of the utility of the model is provided. In the first section the design of the empirical application is described. In the second section the results of the empirical application are discussed.

Design of the Empirical Application

Research Design

The application was a survey study consisting of three phases. In the first phase, multiple regression analysis

was used to estimate the production function and the budget equation. In the second phase, mathematical programming was used to maximize the production function subject to various constraints. The constraints included the budget equation and the bounds of observed variation in the independent variables as well as other constraints deemed appropriate by the investigator. In the third phase, the budget equation was incremented several times and the entire system was resolved using mathematical programming.

It must be stressed that the absolute effect of schools on outputs is not being assessed, but rather the effect of schools on variation in output levels. No doubt schools have an overall baseline effect (i.e., school factors are important for a certain basic level of output). Thus, the analysis did not deal with the effects of "no program" as compared with having a program.¹ Also, the analysis did not consider the question of technical efficiency. This did not invalidate the results of the analysis for the policy-maker. Knowing which type of inputs seem associated with success is important for suggesting reallocation to best affect outcomes within the present technology.

¹As an example of the effects of "no program," the City of New York reported a loss of two months in reading level for the year 1968-69 owing to the two-month teachers' strike. See Mosteller and Moynihan, p. 27.

All research is plagued by the présence of confounding variables. In survey-type research, the inability to manipulate independent variables keeps one from making causal assertions. As an example, an assessment of the potential effects of school resources would have to take into account the possibility that the apparent effects of family background are actually the results of advantaged children attending better-than-average schools. The advantage of multiple regression analysis is that it controls confounding variables by entering them into the regression equation. It is the responsibility of the investigator to anticipate the possible sources of bias as fully as he can and to enter them in the regression analysis. What can never be known is whether or not all such influences have been accounted for. Thus, the results of analysis must always describe associated effects of the independent variables.

Data Base

Data were used for the purpose of illustrating the workability of the model. This differs from the normal usage of data in educational research where the intent is to generalize sample results to a specific population. This was not the goal of the study. It has been stated

previously that the major shortcoming of most studies where mathematical models have been developed is the absence of empirical testing of the proposed model. Yet, this is a crucial stage, for it is at this point that necessary questions regarding the types of data available and the existence of appropriate computational algorithms determine in fact whether or not the model can be made operational. It is one thing to conceptualize the process of education and another to carry this conceptualization through the stages of data collection and data analysis to the point where practical implications can be realized in quantitative statements of policy.

While the above is not intended to grant complete license in the use of the data, it explains why the use differed somewhat from the usual requirements associated with sample usage.

Concurrently, this is allied with the stated intention to study a complete population. The statistical difficulties that can be avoided by studying a complete population have been enumerated in the previous chapter. In estimating the production function the sampling unit is either at the level of state, district, school, or the individual student. In all but the last, the investigator is generally concerned with a finite population of only several hundred. The

significance of this fact was elaborated by Glass and Stanley:

If the ratio of population size to sample size is larger than 100, the techniques appropriate to making inferences to finite populations and those appropriate for infinite populations give essentially the same results. It is customary to use statistical techniques based on the assumption that infinite populations are being sampled whenever the population is reasonably large . . . and the sample from the population does not constitute an appreciable proportion of the population.²

Blalock stated that where interest centers on the regression coefficient in a regression analysis, the sample size should be at least 150.³ Putting the two together implies that in most instances where the population is finite (1,000 or less), it would be better to consider the entire population. For purposes of model illustration, the investigator had access to what was considered the best available data at the present time. The data were collected in a study of school productivity under the direction of the National Educational Finance Project (NEFP).⁴ The study

²Gene V. Glass and Julian C. Stanley, Statistical Methods in Education and Psychology (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1970), p. 241.

³Blalock, p. 187.

⁴See Scott N. Rose, A Study to Identify Variables to Predict Local School District Productivity in Two States (Ed.D. dissertation, University of Florida, 1972).

was based on a sample of 181 school districts in a given state. Due to the unique purposes for which the data were used, the sample cannot be considered to be a representative sample of school districts throughout the state. However, the extremes are well represented in the sample. As Hanushek and Kain stated:

For a study of the educational production process, it is more important to obtain wide variation in educational practice and experience than to have a representative sample of the population of schools or students.⁵

Description of the Variables

Variables were selected from the NEFP data that were descriptive of both in-school and out-of-school factors of each school district. Although there have been many variables identified in related research the variables selected for this study were variables used previously by researchers to examine correlates to school performance or represented variables that were selected by the Research Staff of the National Educational Finance Project as potential predictors of school productivity.

⁵Eric A. Hanushek and John F. Kain, "On the Value of Equality of Educational Opportunity as a Guide to Public Policy," On Equality of Educational Opportunity, p. 118.

Data for the following variables were compiled from available records at the state department of education for a given state, except for variables x_1 through x_3 :⁶

Income per-pupil (x_1)--Data for x_1 were taken directly from Personal Income by School Districts in the United States.

Income under \$3,000 (x_2)--The percentage of gross incomes less than \$3,000 was computed by totaling the number of tax returns per district and dividing this total into the number of returns reporting gross income less than \$3,000.

Income over \$10,000 (x_3)--The percentage of gross incomes over \$10,000 was computed by totaling the number of tax returns per district and dividing this total into the number of returns reporting gross incomes over \$10,000.

ESEA Title I Pupils (x_4)-- X_4 was computed by forming a ratio of numbers of pupils eligible for Title I programs to pupils in ADM. ADM was used as the denominator rather than ADA because the attendance habits of the two groups (ESEA Title I Pupils and total pupils) would not necessarily be the same.

⁶ Data for variables $x_1 - x_3$ were taken from Personal Income by School Districts in the United States, Dewey H. Stollar and Gerald Boardman (Gainesville, Florida: National Educational Finance Project, 1971).

Minority enrollment (x_5)--The district percentages of pupils enrolled, during 1968-69, that were nonwhite, Spanish speaking, Oriental or American Indian were obtained directly from the state department of education.

Attendance (x_6)-- X_6 was calculated by forming a ratio of ADA to ADM. The ADA and ADM were 1968-69 school year figures.

Future training (x_7)--The percentage of graduates receiving post high school education was computed by forming a ratio of the number of 1969 graduates entering future training to total 1969 graduates.

Size of school district (x_8)--ADM for the 1968-69 school year was used as the indicator of school district size.

Percentage enrolled (x_9)--The percentage of children age 5-17 enrolled in public school was calculated from information contained in the fall 1968 school census report filed by each district with the state department of education.

Transportation cost (x_{10})--Transportation cost per pupil was computed by dividing the 1968-69 school year cost for transportation by the number of pupils in ADA.

Local fiscal effort (x_{11})--Local fiscal effort was determined by forming a ratio of local revenue per pupil

in ADA to the adjusted gross income per pupil in ADA. Local revenue per ADA was computed by dividing total local revenue by the number of pupils in ADA during the same year.

Expenses of instruction (x_{12})-- X_{12} was computed by taking the percentage of total current expense disbursed for instruction during the 1968-69 school year.

Longevity experience (x_{13})-- X_{13} was calculated by forming a ratio of teachers with 20 or more years of experience to the total number of teachers for the 1968-69 school year.

Teacher preparation (x_{14})-- X_{14} was computed by forming a ratio of teachers with less than four years' training to the total number of teachers for the 1968-69 school year.

Teacher experience (x_{15})-- X_{15} was calculated by forming a ratio of teachers with less than five years' experience to the total number of teachers for the 1968-69 school year.

Advanced preparation (x_{16})-- X_{16} was computed by forming a ratio of teachers with either an advanced degree or 30 hours of professional training beyond their bachelor's degree to the total number of teachers for the 1968-69 school year.

Median teacher salary (x_{17})--Variable x_{17} was the 1968-69 median teacher salary for each school district.

Average class size (x_{18})--Average class size for the 1968-69 school year was determined by dividing the number of district pupils in ADA by the number of classroom teachers in the district.

Pupil-support personnel ratio (x_{19})--The pupil-support personnel ratio was calculated by forming a ratio of district pupils in ADA during the 1968-69 school year to the number of certified non-teaching personnel employed in the district for the same year.

Expenses for transportation (x_{20})-- X_{20} was computed by taking the percentage of total current expense disbursed for transportation during the 1968-69 school year.

Median reading achievement (x_{21})-- X_{21} was the median score for the school district during 1969 on a standardized reading achievement test for sixth-grade pupils developed by the state department of education.

Average daily attendance (x_{22})-- X_{22} was the average daily attendance for the school district for the 1968-69 school year.

Total current expenditure (x_{23})-- X_{23} was the total current expenditure for the school district for the 1968-69 school year.

Instructional expenditures per-pupil (x_{24})-- X_{24} was computed by forming the ratio of expenses of instruction to ADA (x_{22}).

Table 1 lists the variables described above. Also, a mnemonic label is assigned to each variable to expedite future reference.

Discussion of the Variables

The particular units in which one defines each of the variables used in a study determines the level of data treatment. Above, the observations are measures on the variables at the district level. Variables defined as aggregate amounts per district lead to an estimation of a state production function. If interest centers on a production function at the district level, then the unit of measurement becomes the school. If interest centers on a production function at the school level, then the individual student or classroom must become the unit of measurement. At the state level, unaggregated units of measurement could be used, that is, one could use the individual student or school as the unit of measurement. The necessity for using these units of measurement would depend upon the ratio of between district variation to within district variation. No doubt, aggregated data will contain less variance, but unless the ratio differs radically from one, no great harm is seen.

TABLE 1

VARIABLES POSTULATED AS PREDICTORS OF LOCAL SCHOOL
DISTRICT PRODUCTIVITY FOR A GIVEN STATE

No.	Variable Title ^a	Description
1	Income Per-Pupil (INC/PUP)	Adjusted gross income per-pupil in ADA
2	Incomes under \$3,000 (INC < 3)	Percentage of gross incomes less than \$3,000
3	Incomes over \$10,000 (INC > 10)	Percentage of gross incomes over \$10,000
4	ESEA Title I Pupils (ESEAPU)	Percentage of pupils eligible for Title I instruction under ESEA
5	Minority Enrollment (MINENR)	Percentage of pupil enrollment that is nonwhite, Spanish speaking, Oriental or American Indian
6	Attendance (ADA/ADM)	Ratio of ADA to ADM
7	Future Training (FUTRAN)	Percentage of graduates receiving post high school education or training
8	Size of School District (ADM)	Average daily membership

TABLE 1 (CONTINUED)

No.	Variable Title ^a	Description
9	Percentage Enrolled (ENRPUB)	Percentage of children age 5-17 enrolled in public school
10	Transportation Cost (TRAN/P)	Transportation cost per-pupil in ADA
11	Local Fiscal Effort (LFISEF)	Ratio of local revenue per-pupil in ADA to adjusted gross income per-pupil in ADA
12	Expenses for Instruction (TCEI)	Percentage of total current expenditure funded for instruction
13	Longevity Experience (EXP > 20)	Percentage of teachers with 20 or more years of experience
14	Teacher Preparation (PREP < 4)	Percentage of teachers with less than four years of training
15	Teacher Experience (EXP < 5)	Percentage of teachers with less than five years of experience
16	Advanced Preparation (ADPREP)	Percentage of teachers with bachelor's (BA) plus 30 hours of additional training or an advanced degree
17	Median Teacher Salary (MEDSAL)	Median annual teacher salary for the school district

TABLE 1 (COMPLETED)

No.	Variable Title ^a	Description
18	Average Class Size (AVCLSI)	Ratio of pupils in ADA to the number of classroom teachers
19	Pupil-Support Personnel Ratio (PUSUPR)	Ratio of pupils in ADA to the number of certified non-teaching personnel of the district.
20	Expenses for Transportation (TCET)	Percentage of total current expenditures funded for transportation
21	Median Reading Achievement (MEDACH)	Median score on standardized reading achievement test for sixth-grade pupils
22	Average Daily Attendance (ADA)	Average daily attendance for the school district
23	Total Current Expenditure (TCE)	Total current expenditure for the school district
24	Instructional Expenditures Per-Pupil (TCEI/P)	Ratio of expenses of instruction to ADA

^aThe expression in parentheses is a mnemonic label for the variable title

The reason for restricting the units of measurement at a given level to those discussed above follows. Statistically, it can be shown that the ability to predict with a multiple regression equation is greatest when using the mean values of the independent variables. As an example, in estimating a state production function based on data at the district level, the state mean on the criterion variable could be predicted best by using state means on all independent variables. For educational policy-makers operating at the state level, this would be the appropriate level of data treatment.

It is important to realize that since information about school resources was gathered only on a district-wide basis, the school resource inputs cannot account for any of the differences among schools or among students within the same school district. Therefore, the percentage of total variance that lies among school districts is a kind of upper limit on the amount of variance for which school inputs can account. On the other hand, by working with aggregated scores for each variable, the Law of Large Numbers can be used with advantage, that is, greater accuracy and less measurement error will be obtained with macroscopic variables. Thus in using aggregative statistics the data may not be adequate for decisions about individual

schools, but they will be adequate for deciding policy for school districts as a whole. At any rate, attention must be given to the ecological fallacy, that is, the relationships that were demonstrated hold only at the aggregate level and not necessarily for individual schools.

The only output included in the present list of variables was the sixth-grade median reading achievement score for the district (x_{21}). While there is some concern over whether or not cognitive skills are the most useful outcomes of schooling, knowing something meaningful about success in the area of cognitive scores is far superior to knowing nothing at all, and this knowledge should allow some general inferences that would be most helpful in pointing general policy directions.

Given that inputs in the educational process include community and student as well as school inputs, the inputs observed in the sample data appear to strike a balance among the three types of inputs. The socio-economic variables can be seen as measures of both student and community inputs. Persons of different socio-economic status (SES) face different kinds of life situations and in adapting to these they may develop different sets of values and life styles. In short, SES variables serve as proxies for a variety of values, attitudes, and motivations related to academic

performance. The same can be said of community inputs, for which family values and life styles are manifested in an aggregate sense for community life.

The school variables appear to measure mainly the characteristics of teachers and the instructional program as reflected in instructional expenditures, average class size, and pupil-support personnel ratio. Separate programs (reading, science) as they exist in the schools are not differentiated. It also becomes apparent that in estimating the prices of the various school inputs, the most serious source of bias is revealed. It can be argued that living costs are higher in larger cities and that a part of the salary differential between large and small school districts may be considered a corrective of local price variation. In short, a cost of living index may be needed to help explain the variation in per-pupil expenditures across districts.

In summary, the variables from the sample data may be considered to be less than an ideal set. It would seem that a relatively small number of school variables have been considered of what is potentially available. This is not considered a problem if it is kept in mind that the primary goal was to simply illustrate the model using the best available data and it was not crucial if analysis was restricted only to a limited area of educational policy.

The Data Analysis Plan

The first step in the data analysis involved estimating the production function and the budget equation using multiple regression. A program from Biomedical Computer Programs (BMD) was used.⁷ The particular program is identified as BMD02R - Stepwise Regression. The program computes a sequence of multiple regression equations in a stepwise manner. At each step one variable is added to the regression equation. The variable added is the one which has the highest partial correlation with the dependent variable partialled on the variables which have already been added. In addition, variables can be forced into the regression equation. Regression equations with or without a regression intercept may be selected. This program proved highly satisfactory for several reasons. First, of the variables containing redundant information (highly related), stepwise regression selected only the ones that were optimally related to the dependent variable, omitting the remaining variables from the equation. This helped to avoid the partialling fallacy. Secondly, since most school variables were wanted in the equation, these variables were forced.

⁷Biomedical Computer Programs, ed. by W. J. Dixon (Berkeley: University of California Press, 1971).

Third, the stepwise procedure allowed the contribution to the multiple R of each variable as it entered the equation to be viewed (i.e., what has been gained by allowing this variable to come into the equation).

The process of estimating these two equations was exploratory in nature. Thus, as a first step all linear terms were considered, some being forced into the equation. Secondly, all quadratic and cubic terms were tested. Then selected interaction terms were tested. It was not possible to examine all interaction terms simultaneously. In a sample of 181, the maximum number of terms that the regression equation could have was 180. Yet with 20 independent variables there were 190 possible interactions. Thus analysis was limited to those interactions that were logically viewed as having potential impact on school achievement (community-school and school-school interactions). Finally, the analysis ended with nonlinear functions consisting of certain forced linear terms and any other terms which made a significant contribution to the multiple R (linear, higher degree or interaction terms).

Given the production function and the budget equation, the second major stage of the data analysis was solving a mathematical programming problem in which the production function was the objective function and the constraints

included the budget equation and bounds on the observed variation in the independent variables. A program for solving nonlinear programming problems comes from the SHARE Program Library.⁸ The particular program is SDA 3189-SUMT. The purpose of this program is to solve nonlinear mathematical programming problems where the objective function and constraints may be nonlinear. The program uses SUMT (Sequential Unconstrained Minimization Technique) to solve the mathematical programming problem. Users must supply a subroutine to read in the problem data and three subroutines to evaluate the problem functions, and their first and second partial derivatives. If the objective function is not concave, then it is necessary to start the algorithm at various points in the feasible domain of solutions.

Results of the Empirical Application

In analyzing the data, it was recalled from the statement of the problem that three problems were to be solved.

1. To determine the production function that describes the input-output relationship between selected variables;
2. To determine the optimal combination of inputs to maximize output subject to the budget and other constraints;

⁸The SHARE Programming Library consists of a central file and documentation system available at most IBM installations.

3. To determine the optimal combination of inputs to maximize output, subject to certain constraints, given increments in the budget constraint.

To solve (1) multiple regression was used. Also, the budget equation was estimated using multiple regression. Problems (2) and (3) were solved using mathematical programming.

Basic Data Description

Repeatedly, the necessity for knowing the ranges of each of the variables in the observed data has been indicated. This was essential since predictions cannot be extrapolated beyond the range of observed variation. At the same time, the means and standard deviations of each variable are listed to provide additional information regarding the variability of the data. This information is shown in Table 2. It was believed that these data provided the widest possible variation in existing data for a given state.

Estimating the Production Function

A multiple regression equation was developed using median reading achievement (x_{21}) as the dependent variable and all of the other variables as independent variables except for ADM (x_8). This represented a choice between ADM (x_8) and ADA (x_{22}). ADA was preferred over ADM since

TABLE 2

BASIC DATA DESCRIPTION

No.	Variable Title	Mean	Standard Deviation	Maximum	Minimum
1	Income Per-Pupil	11,456.35	8,217.34	65,797.00	893.00
2	Incomes under \$3,000	29.27	5.03	43.20	20.40
3	Incomes over \$10,000	22.53	12.74	56.70	3.60
4	ESEA Title I Pupils	4.15	5.20	38.50	0.0
5	Minority Enrollment	7.85	15.56	95.60	0.0
6	Attendance	93.90	1.55	96.60	87.60
7	Future Training	64.53	15.38	98.00	17.60
8	Size of School District	10,084.36	79,023.75	1,063,116.00	332.00
9	Percentage Enrolled	86.47	10.41	99.00	47.00
10	Transportation Cost	65.48	26.50	144.22	7.70
11	Local Fiscal Effort	5.59	3.94	32.10	0.30
12	Expenses for Instruction	64.42	3.45	79.40	55.20

13	Longevity Experience	17.57	8.92	52.60	1.70
14	Teacher Preparation	5.87	5.46	32.10	0.0
15	Teacher Experience	37.38	9.57	60.60	14.70
16	Advanced Preparation	52.49	12.05	84.30	22.20
17	Median Teacher Salary	9,149.64	1,157.65	13,750.00	6,950.00
18	Average Class Size	18.50	1.66	22.20	12.30
19	Pupil-Support Personnel Ratio	136.55	82.20	898.30	44.90
20	Expenses for Transportation	5.64	2.33	11.70	0.60
21	Median Reading Achievement	43.07	6.86	55.48	28.73
22	Average Daily Attendance	9,180.50	70,852.56	953,107.00	316.00
23	Total Current Expenditure	11,490,572.00	91,340,928.00	1,229,253,000.00	361,919.00
24	Instructional Expenditures Per-Pupil	764.51	139.32	1,287.90	554.13

other variables were computed on the basis of ADA rather than ADM. In estimating the production function it was imperative that all of the relevant school inputs be represented in the equation. This was accomplished by forcing-in such variables. In this case variables x_{12} - x_{19} were forced into the equation. It is noted that variables x_{10} (transportation cost per pupil) and x_{20} (percent of total current expenditures funded for transportation) deal with school policy. However, these variables were not subject to manipulation by school authorities (i.e., no one expects the school board to move the students closer to their schools). Consequently, they were left free to enter the equation if they had significant impact on school achievement. Also, ADA (x_{22}) was forced into the equation. The rationale for this choice was that the investigator wanted ADA represented in the regression equation so that questions regarding the relationship of district size and achievement, constrained by costs, could be considered.

In testing for nonlinearities in the independent variables considered, all square terms and cubic terms and certain interaction terms were tested (i.e., tested to see if significant contributions to the variance in achievement exists). In regression analysis, however, the number of terms must always be less than the number of cases in

the data. This made it necessary to conduct two separate "runs." In the first "run" all square and cubic terms were tested. Those that proved to be insignificant were omitted from further consideration in the second "run." In the second "run," selected interaction terms were considered. Again, the possible number of interaction terms outnumbered the number of cases. Logically, though, consideration need only be given to those possible interaction terms which can reasonably be assumed to have an effect on achievement. In this case, interest centered on the interactions between school and student background variables and the interactions between certain school variables. Table 3 lists the interaction terms that were tested.

TABLE 3

INTERACTION TERMS TESTED FOR THE PRODUCTION FUNCTION^a

Interactions Between Student-Background and School Variables			
x_2x_{13}	x_3x_{13}	x_4x_{13}	x_5x_{13}
x_2x_{14}	x_3x_{14}	x_4x_{14}	x_5x_{14}
x_2x_{15}	x_3x_{15}	x_4x_{15}	x_5x_{15}
x_2x_{16}	x_3x_{16}	x_4x_{16}	x_5x_{16}
x_2x_{17}	x_3x_{17}	x_4x_{17}	x_5x_{17}
x_2x_{18}	x_3x_{18}	x_4x_{18}	x_5x_{18}
x_2x_{19}	x_3x_{19}	x_4x_{19}	x_5x_{19}

TABLE 3 (CONTINUED)

Interactions Between School Variables			
$x_{13}x_{14}$	$x_{14}x_{15}$	$x_{15}x_{16}$	$x_{16}x_{17}$
$x_{13}x_{16}$	$x_{14}x_{17}$	$x_{15}x_{17}$	$x_{16}x_{18}$
$x_{13}x_{17}$	$x_{14}x_{18}$	$x_{15}x_{18}$	$x_{17}x_{18}$
$x_{13}x_{18}$			

^aSee Table 1 for a description of variable codes

To determine whether a variable that was tested made a significant contribution involved making a choice as to the significance level to be employed. For those terms that were free to come into the equation (i.e., non-forced), it was felt that they should enter only if a noteworthy contribution was being made. The investigator observed that by setting the F-to-enter at 6.80 ($.99F_{1,160} = 6.80$), variables coming into the equation made an addition to the multiple R of generally 1 or more percent. It was felt that in latter steps of the regression analysis, any variable that could make a contribution of 1 or more percent should enter the equation.

Table 4 gives the results of the analysis for estimating the production function. The equation appears to predict

TABLE 4

REGRESSION COEFFICIENTS FOR THE PRODUCTION FUNCTION
(DEPENDENT VARIABLE: READING ACHIEVEMENT-- X_{21})

No.	Variable Title	Coefficient ^a	F-to-Enter ^b	F-to-Remove ^c	
<u>Linear Terms</u>					
3	Incomes over \$10,000	0.150	33.95	13.26	(2) ^d
6	Attendance	-52.807	8.91	14.35	(2)
7	Future Training	0.093	10.35	14.35	(2)
12	Expenses for Instruction	4.292	14.26	6.87	(3)
13	Longevity Experience	-0.042	3.13	1.04	(3)
14	Teacher Preparation	-1.511	5.09	11.12	(3)
15	Teacher Experience	-0.001	2.80	0.00 ⁺	(3)
16	Advanced Preparation	0.120	40.23	10.53	(3)
17	Median Teacher Salary	0.002 ^e	3.19	0.00 ⁺	(3)
18	Average Class Size	0.220	3.39	1.75	(3)
19	Pupil-Support Ratio	0.011	3.41	9.63	(3)
22	Average Daily Attendance	-0.004 ^e	5.51	0.71	(3)
<u>Quadratic Terms</u>					
6	X_6^2	0.291	133.39	15.16	(2)
12	X_{12}^2	-0.035	7.51	7.51	(2)
<u>Interaction Terms</u>					
	X_4X_{19} ^f	-0.002	19.34	10.55	(2)
	X_5X_{18} ^g	-0.005	9.42	16.35	(2)
	$X_{14}X_{17}$	0.016	11.88	8.80	(2)
<u>Constant Term</u>					
		2282.167			

$$R^2 = 0.79$$

^aThe regression coefficient is the b weight (unstandardized partial regression coefficient)

^bF test of significance of a single variable in a stepwise regression at the step of entry into the equation

^cF test of significance of a single variable in a stepwise regression after the final step of the regression

^d2 = free variable; 3 = forced variable

^eVariables X_{17} , X_{22} were scaled so that truncation errors would not occur. X_{17} : 1 unit = \$100; X_{22} : 1 unit = 1,000 pupils

^f X_4 : ESEA Title I Pupils

^g X_5 : Minority Enrollment

fairly well as evidenced by $R^2 = 0.79$. The interpretation of associated effects is not as simple in the case where nonlinear terms are involved. One must use partial derivatives to determine the effects. As an example:

$$\frac{\partial x_{21}}{\partial x_6} = -52.807 + 2(0.291)x_6.$$

The effect of x_6 on x_{21} depends on the level of x_6 . If the effect at the mean value of x_6 (93.9) is considered, then

$$\frac{\partial x_{21}}{\partial x_6} = 1.843.$$

This states that an increase of 1 percent in ADA/ADM (x_6) at the mean level of x_6 was associated with an increase in reading achievement (x_{21}) of 1.843 units.

While the designated regression coefficients could provide pages of discussion, it was not the primary goal to discuss all of the pertinent interpretations. However, a few are worthy of discussion since they highlight the necessity for a nonlinear model. Looking at the regression coefficient for average class size (x_{18}), a positive relationship is seen. The zero order correlation coefficient between average class size (x_{18}) and achievement (x_{21}) was 0.036. It would seem that increases in class size were associated with increases in achievement. Of importance, though, was the interaction of class size with minority

enrollment (x_5). Increases in the percentage of minority students were associated with decreases in achievement. Thus, if minorities constituted 50 percent of the school population, an increase of one unit in class size was associated with a decrease of 0.030 units in achievement. This is where the zero order correlation coefficient can be misleading since it did not reveal the interaction that was occurring between class size and minority enrollment.

The regression coefficient for the percentage of teachers with less than four years' training (x_{14}) was as expected ($b = -1.511$). However, this percentage interacted with median salary (x_{17}). The interaction coefficient stated that increases in median salary for a given percentage of x_{14} were associated with increases in achievement. In fact, for $x_{17} = \$10,000$, an increase of 1 percent in x_{14} was associated with an increase of 0.099 units in achievement. As in all such cases where trouble is suspected, recourse was made to a frequency distribution. Table 5 reveals the results of such analysis. What was observed was that as the salary level increased the range of x_{14} became more and more restricted. Thus when

$$100 \leq x_{17} < 110,$$

then $0 \leq x_{14} \leq 12.5$.

If the interaction is to be meaningful when $x_{17} = 105$, then x_{14} must be in the interval from 0 to 12.5. This fact became more crucial when the stage of programming analysis was considered.

TABLE 5

AN EXAMINATION OF THE INTERACTION BETWEEN THE PERCENTAGE
OF TEACHERS WITH LESS THAN FOUR YEARS'
TRAINING (x_{14}) AND MEDIAN TEACHING SALARY (x_{17})

Interval No. ^a	x_{17} ^b	x_{14} ^c	No. of Cases
1	[69.5, 80) ^d	[2 , 32.1]	22
2	[80 , 90)	[0 , 25]	73
3	[90 , 100)	[0 , 23.5]	48
4	[100 , 110)	[0 , 12.5]	25
5	[110 , 120)	[0 , 3]	10
6	[120 , 130)	[0.9, 1]	2

^aGiven an interval for x_{17} , such as [69.5, 80), the interval of corresponding values for x_{14} was determined. This means that as x_{17} assumed values in the interval [69.5, 80), the range of x_{14} was restricted to the interval [2, 32.1].

^b x_{17} was scaled. 1 unit = \$100

^c x_{14} represented a percentage

^d[,) means that the interval includes the number on the left, but not on the right.

In summary, the analysis estimated a production function which predicted very well and most of the parameters conformed to expectations.

Estimating the Budget Equation

Basically, the same procedure was followed for estimating both the budget equation and the production function. The total current expenditure for instruction per-pupil (x_{24}) was the dependent variable. A per-pupil cost variable was preferred since using total cost leads to scaling difficulties. Since most of the school variables used in the study were considered instructional expenditures it was felt that per-pupil instructional expenditures was a better choice for a dependent variable than per-pupil total current expenditures (x_{23}).

The same variables were forced or allowed to be free as in the previous analysis with some exceptions. Since total current expenditure (x_{23}) was highly related to x_{24} it was omitted from the analysis. The use of this variable would not be meaningful, since x_{23} was not a component of x_{24} . Rather, the reverse is true. Likewise, the interactions of school variables were not necessarily meaningful in comparing costs, and were thus omitted from the analysis. Given the remaining variables, as before, square terms, cubic terms and interaction terms were tested. The results are given in Table 6. Other than linear terms, only three square terms were significant (x_5^2 , x_{16}^2 , x_{17}^2). No cubic terms or interaction terms entered the equation.

TABLE 6

REGRESSION COEFFICIENTS FOR THE BUDGET EQUATION
(DEPENDENT VARIABLE: INSTRUCTIONAL
EXPENDITURES PER PUPIL-- X_{24})

No.	Variable Title	Coefficient ^a	F-to-Enter ^b	F-to-Remove ^c	
<u>Linear Terms</u>					
10	Transportation Cost	7.352	13.86	277.51	(2) ^d
12	Expenses for Instruction	7.730	41.30	51.33	(3)
13	Longevity Experience	-0.364	2.19	0.72	(3)
14	Teacher Preparation	-0.674	27.70	0.78	(3)
15	Teacher Experience	0.409	11.06	0.72	(3)
16	Advanced Preparation	-8.817	6.22	17.51	(3)
17	Median Teacher Salary	-14.256 ^e	197.31	15.30	(3)
18	Average Class Size	-6.662	99.21	8.54	(3)
19	Pupil-Support Ratio	-0.032	12.97	0.78	(3)
20	Expenses for Transportation	-85.886	136.10	235.80	(2)
22	Average Daily Attendance	-0.358 ^e	12.77	36.82	(3)
<u>Quadratic Terms</u>					
5	X_5^2 ^f	0.021	13.26	51.88	(2)
16	X_{16}^2	0.091	21.20	20.77	(2)
17	X_{17}^2	0.090	22.06	22.06	(2)
<u>Constant Term</u>		1125.488			
$R^2 = 0.94$					

^aThe regression coefficient is the b weight (unstandardized partial regression coefficient)

^bF test of significance of a single variable in a stepwise regression at the step of entry into the equation

^cF test of significance of a single variable in a stepwise regression after the final step of the regression

^d2 = free variable; 3 = forced variable

^eVariables X_{17} , X_{22} were scaled so that truncation errors would not occur. X_{17} : 1 unit = \$100; X_{22} : 1 unit = 1000 pupils

^f X_5 : Minority Enrollment

The equation does appear to explain most of the variation in per-pupil instructional expenditures across districts with a multiple $R^2 = 0.94$.

Perhaps the most noteworthy result of the analysis was the lack of a square term for ADA (x_{22}). Per-pupil cost was not quadratic as many have hypothesized. Rather, the larger the district, the lower the per-pupil cost. Given the range in ADA (316 to 953,107), which was about as wide as can be, there appeared to be no optimal size for per-pupil costs. The F-to-Enter for x_{22}^2 at the last step in the analysis was 0.8893. Variables x_{16} (ADPREP) and x_{17} (MEDSAL) had significant square terms. Thus the cost of increasing either one of these variables depended on their levels. The larger x_{16} was, the more it cost to increase it by one additional unit. Exactly what implicit factors were operating to cause this was not clear. One thing was clear, by examining the F-to-remove ratios, the square terms made a significant contribution to the multiple R and thus a parabola provided a better fit to the observed data than a straight line.

In summary, it appears that the budget equation estimated the prices of the school inputs reasonably well. Furthermore, the coefficients gave the price of each variable while other variables were held constant. This was a necessary requirement for the programming analysis.

Formulation of the Mathematical Programming Problem

In solving a mathematical programming problem the educational production function becomes the objective function and the constraints include the budget equation as well as bounds on the observed variation of the independent variables. Model specification of the production function requires community and student inputs as well as school inputs. However, community and student inputs cannot be manipulated by school authorities. These inputs cannot be considered as variables in the programming analysis. Rather, the objective is to maximize the production function for a given set of community and student inputs. More specifically, given the production function:

$$A_{it} = g(C_i^{(t)}, S_i^{(t)}, I_i, P_i^{(t)}),$$

for a given set of community, peer-group, and individual inputs, the above can be transformed to:

$$A_{it} = g(k, S_i^{(t)})$$

where k is a constant. Thus for a given set of school inputs, the problem is to

$$\begin{aligned} &\text{Maximize } A_{it} = g(k, S_i^{(t)}) \\ &\text{subject to } \Sigma f_i(k, S_i^{(t)}) \leq B_i. \end{aligned}$$

Since the variables were macroscopic (i.e., level of data treatment was the district), individual inputs were

deleted from the model. Interest centered on the analysis of educational policy for the population as a whole. Thus, for community and student inputs the mean values of the relevant variables were used. Statistically, this had an enormous benefit, since in the production function the mean level of achievement for the complete population could be predicted perfectly (i.e., no residual error occurs).

Thus, the school inputs ($x_{12} - x_{19}$) should become the variables in the programming analysis, all other variables assuming constant values equal to the population means. Further reflection revealed, however, that variable x_{12} (percent of total current expenditure for instruction) was not manipulable in this analysis. All other school variables ($x_{13} - x_{19}$) related to instructional resources and the dependent variable in the budget equation was total instructional expenditure per-pupil (x_{24}). Thus, ways of reallocating resources in the instructional category with a given budget for instructional expenses were being considered. This does not change the percentage of total current expenditure going to instruction, since the instructional budget remains the same. Thus in the analysis x_{12} remained constant.

Another difficulty centered on the interaction term, $x_{14}x_{17}$ (PREP•MEDSAL). Preliminary data analysis revealed

that the maximum observed value for x_{14} (32.1) should be used with x_{17} set at 122.5. Nevertheless, an examination of the frequency distribution did not support this conclusion since the range of x_{14} became more and more restricted as x_{17} increased. This problem was discussed earlier in the chapter. The problem was resolved by making x_{14} constant. A number of reasons can be advanced for setting $x_{14} = 0$. While teachers with less than 4 years' training may be skillful at teaching reading, it can be questioned whether or not their training has been sufficient for them to function as well in helping students attain other outcomes, both cognitive and affective. It is doubtful that beginning teachers with less than four years' training would be prepared to teach the new science and mathematics programs that are now a part of elementary curriculum. In the past, school districts have not always been able to employ all fully certified staff. Thus, the value of $x_{14} = 5.87$, which was the mean value observed in the population, was also considered.

School district size, as determined by average daily attendance (ADA - x_{22}), was potentially manipulable to the extent that consolidation of school districts can and should occur. Earlier no optimal scale of economy with respect to district size and per-pupil cost was reported.

It was observed that the larger the district the lower the per-pupil cost. However, achievement was negatively related to district size ($b = -0.004$). It is possible, therefore, that there would be an optimum size for the school district for maximizing achievement, constrained by cost. Preliminary data analysis suggested the unrealistic figure of approximately 700,000, unrealistic because other outcomes would probably be negatively affected and in most instances it would not be feasible geographically to reorganize districts on such a scale. Thus x_{22} can be set at various levels and the effects of different district sizes on achievement and the distribution of resources for a given per-pupil budget can be observed.

The foregoing suggests that analyses be conducted at various levels of x_{14} ($PREP < 4$) and x_{22} (ADA). In addition to the constraints previously described it was also observed that the sum of variables x_{13} ($EXP > 20$) and x_{15} ($EXP < 5$) must always be less than 100 and that the same held true for x_{14} ($PREP < 4$) and x_{16} (ADPREP). With the above in mind, then, the mathematical programming problem to be solved reduced to the following:

$$\text{Let } x_{21} = \phi(x) + 32.744 - 1.511x_{14} - 0.004x_{22}.$$

Maximize

$$\phi(x) = -0.042x_{13} - 0.001x_{15} + 0.120x_{16} + 0.002x_{17} \\ + 0.016x_{14}x_{17} + 0.179x_{18} + 0.002x_{19}$$

subject to:

$$\text{I.} \quad (1) \quad -0.364x_{13} + 0.409x_{15} - 8.817x_{16} - 14.256x_{17} \\ - 6.662x_{18} - 0.032x_{19} + 0.091x_{16}^2 + 0.090x_{17}^2 \\ \leq -862.286 + 0.674x_{14} + 0.358x_{22}$$

(Budget Equation)

II.	(2)	$x_{13} + x_{15} \leq 100$	<u>Scaled Variables</u>
	(3)	$x_{14} + x_{16} \leq 100$	x_{17} : 1 unit = \$100
III.	(4)	$1.7 \leq x_{13} \leq 52.6$	x_{22} : 1 unit = 1,000 pupils
	(5)	$14.7 \leq x_{15} \leq 60.6$	
	(6)	$22.2 \leq x_{16} \leq 84.3$	<u>Constants</u>
	(7)	$69.5 \leq x_{17} \leq 137.5$	x_{14}, x_{22}
	(8)	$12.3 \leq x_{18} \leq 22.2$	
	(9)	$44.9 \leq x_{19} \leq 898.3$	

Since $x_{21} = \phi(x) + \text{constant}$, it follows that in maximizing $\phi(x)$, x_{21} is also being maximized. For that reason programming algorithms never have a constant term in the objective function, since it is not needed and its omission simplifies the programming process. Thus a simple transformation of variables can always eliminate the constant term in the objective function. Of importance was whether

or not the objective function was concave. Given that x_{14} = constant, $\phi(x)$ was a linear function. Linear functions are always concave. Thus $\phi(x)$ was a concave function. This meant that any local optimum determined by the algorithm would also be a global optimum.

Results of the Programming Analysis

In order to expedite the comparisons that were to be made with each "run" of the analysis, Table 7 lists the variables manipulated in the programming analysis along with the mean values observed in the population. Obviously, after each optimization the results can be compared against the mean values.

For the first "run" x_{14} (PREP < 4) and x_{22} (ADA) were set at their mean values. The results were

x_{21}	x_{13}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
48.0	1.7	14.7	73.5	9950	22.2	898.3
	LB	LB	OPT	OPT	UB	UB

LB - Lower bound of the variable

UB - Upper bound of the variable

OPT - Optimal value of the variable ($LB < OPT < UB$).

The analysis suggested that an optimal allocation of resources among the given variables would predict an increase in achievement (x_{21}) of 4.9 units. This represented an increase of 0.71 standard deviations. Experience did not

TABLE 7
VARIABLES MANIPULATED IN THE MATHEMATICAL
PROGRAMMING ANALYSIS

<u>No.</u>	<u>Variable Title</u>	<u>Mean</u>	<u>Standard Deviation</u>
13	Longevity Experience	17.57	8.92
14	Teacher Preparation ^a	5.87	5.46
15	Teacher Experience	37.38	9.57
16	Advanced Preparation	52.49	12.05
17	Median Teacher Salary	9,149.64	1,157.65
18	Average Class Size	18.50	1.66
19	Pupil-Support Ratio	136.55	82.20
21	Median Reading Achievement ^b	43.07	6.86
22	Average Daily Attendance ^a	9,180.50	70,852.56
24	Instructional Expenditures Per-Pupil ^c	764.51	139.32

^aFor each programming problem, X_{14} and X_{22} assumed constant values

^b X_{21} was the variable maximized

^c X_{24} was the budget constraint

seem to count (x_{13} , x_{15}), nor did class size (x_{18}) or the pupil-support ratio (x_{19}). Advanced training (x_{16}) was deemed most important of the variables affecting achievement; salary (x_{17}) counted also. If the salary increments were eliminated for experience, and class size and pupil-support ratios increased to their maximums, enough resources would be released within the existing budget to increase x_{16} (ADPREP) by 1.74 S.D. and x_{17} (MEDSAL) by 0.69 S.D. It should be noted at this point that in a programming analysis, variables are assumed to be independent of each other, that is, a change in one should not automatically cause change in others. Thus, given the existing salary increments for advanced training, an increase in x_{16} (ADPREP) was likely to lead to an increase in x_{17} (MEDSAL). The analysis could not reveal what that change would be. However, increases in x_{17} can be viewed as that accruing beyond that due to x_{16} . To express it another way, the above suggests that the median salary could still be increased by 0.69 S.D. (perhaps by raising the base salary) after increases in median salary due to increases in advanced preparation had been attributed.

Some may suggest that by allowing the pupil-support ratio (x_{19}) to go to its maximum value, other outcomes (most likely affective ones) would be affected. This

could be an instance of increasing one output at the expense of another. With the model, though, minimum standards to be achieved by other outputs can be set. Thus a pupil-support ratio of 136.5 (population mean) may be necessary to maintain minimum standards on other outputs. Consider the second "run," then, where $x_{19} = 136.5$.

x_{21}	x_{13}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
45.7	1.7	14.7	70.1	9670	22.2	136.5
	LB	LB	OPT	OPT	UB	FIXED

By requiring a maximum pupil-support ratio of 136.5, achievement increased by 0.38 S.D. as against 0.71 S.D. previously. More resources were expended to maintain this level for x_{19} , resources that take away from advanced training and salary with consequent effects on reading achievement.

Likewise, one could argue that increasing classroom size to its maximum value would negatively affect other outcomes. Consider then if an average class size of 18.5 (population mean) is maintained.

x_{21}	x_{13}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
44.1	1.7	14.7	65.8	9320	18.5	136.5
	LB	LB	OPT	OPT	FIXED	FIXED

Achievement increased by 0.15 S.D. By "trading off" only on the resources expended for experience, less is available to increase x_{16} and x_{17} . Actually, there is evidence to suggest that decision-makers, given the choice of reducing

class size or increasing salaries, have chosen to increase salaries.⁹ Thus in future "runs" class size assumed its maximum value.

The runs considered so far have had x_{14} ($\text{PREP} < 4$) equal to its mean value. Consider what happened when $x_{14} = 0$:

x_{21}	x_{13}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
46.1	1.7	14.7	75.4	7970	22.2	136.5
	LB	LB	OPT	OPT	UB	FIXED

Comparing this with a previous "run" a greater predicted increase in achievement (0.44 S.D.) was found as expected, that is, decreases in x_{14} ($\text{PREP} < 4$) should be associated with increases in achievement ($b = -1.511$). Looking at x_{17} (MEDSAL), however, a decrease from the mean value of 9,150 was noted. This was due to the interaction term $x_{14}x_{17}$ which was discussed earlier. The effect that salary had on achievement interacted with x_{14} . The greater the percentage of teachers with less than four years training, the more important the level of salary became. However, when $x_{14} = 0$, the b weight for salary was only 0.002, which was small compared to advanced training ($b = 0.120$). No

⁹Herbert J. Kiesling, The Relationship of School Inputs to Public School Performance in New York State (Santa Monica, Calif.: The Rand Corporation, October, 1969), p-4211, p. 24.

wonder then that analysis suggested that more money be spent on x_{16} (ADPREP). One interpretation of the above could be that by reducing the base salary by a given amount, this money could then be expended for teachers with advanced training. As to what the overall median salary would be as a result is not clear. If decision-makers were reluctant to decrease the base salary, then the model allows a lower bound to be set on the salary level and the problem can be resolved.

The effect that district size had on achievement and the distribution of resources was considered. Throughout, it was assumed that $x_{14} = 5.87$ (population mean). Table 8 gives the results of the analysis at various levels of x_{22} (ADA). The table shows that by increasing the size of the district, the per-pupil cost was lowered and hence within a given budget more money could be expended on x_{16} and x_{17} with the consequent effects on achievement. Johns and Morphet concluded that the optimal size of a school district should be approximately 50,000 students.¹⁰ In going from the population mean of 9,180 to 50,000, with the same per-pupil budget, the percentage of teachers with advanced

¹⁰Edgar L. Morphet et al., Educational Organization and Administration (Englewood Cliffs, N.J.: Prentice-Hall, Inc., 1967), p. 174.

TABLE 8

EFFECT OF DISTRICT SIZE ON ACHIEVEMENT AND THE
DISTRIBUTION OF SCHOOL RESOURCES ($x_{14} = 5.87$)

x_{22}	x_{21}^b	x_{13}	x_{15}	x_{16}	x_{17}	x_{18}	x_{19}
1,000	45.7	1.7	14.7	69.6	9,630	22.2	136.5
5,000	45.7	1.7	14.7	69.8	9,650	22.2	136.5
9,180 ^a	45.7	1.7	14.7	70.1	9,670	22.2	136.5
20,000	45.8	1.7	14.7	70.6	9,710	22.2	136.5
30,000	45.9	1.7	14.7	71.2	9,760	22.2	136.5
50,000	46.0	1.7	14.7	72.2	9,840	22.2	136.5
70,000	46.1	1.7	14.7	73.2	9,920	22.2	136.5
100,000	46.3	1.7	14.7	74.6	10,030	22.2	136.5

^a9,180 was the mean value observed in the population

^b43.1 was the mean value observed in the population

training (x_{16}) could be increased by 2.1 percent and the median salary (x_{17}) could be increased \$170 above that due to increases in x_{16} .

To complete the final stage of the data analysis the per-pupil budget was incremented by fixed amounts. It was assumed that $x_{22} = 9,180$ and two levels of x_{14} , namely $x_{14} = 0$ and $x_{14} = 5.87$, were considered. Table 9 gives the results of the analysis. The interpretation of these results presented some problems. First, we note that increasing the per-pupil budget by \$100 had a greater effect on achievement when $x_{14} = 5.87$ than when $x_{14} = 0$ ($x_{21} = 48.1$ as against $x_{21} = 47.2$). That this was so is obscured by the following. At some point in increasing the budget \$100, for the case where $x_{14} = 0$, x_{16} reaches its maximum value. From that point on, only salary (x_{17}) can make a contribution, but its effect was far less than that of x_{16} . That may be why x_{21} was greater when $x_{14} = 0$ than when $x_{14} = 5.87$ and the budget increment was \$0, but was reversed when incremented \$100. This would suggest that the lower half of the table is more easily interpreted than the upper half. At any rate, when $x_{14} = 5.87$, a budget increment of \$100 raised achievement from 45.7 to 48.1 (increase of 0.35 S.D.). In incrementing \$200, achievement increased to 49.7. The rate of increase was

TABLE 9
THE EFFECT OF INCREMENTING THE BUDGET
ON ACHIEVEMENT ($X_{22} = 9,180$)

Increments in Per- Pupil Budget	X_{21}	X_{13}	X_{15}	X_{16}	X_{17}	X_{18}	X_{19}
<u>$X_{14} = 0.0$</u>							
\$0	46.1	1.7 LB ^a	14.7 LB	75.4 OPT	7,970 OPT	22.2 UB	136.5 FIXED ^b
\$100	47.2	1.7 LB	14.7 LB	84.3 UB	10,250 OPT	22.2 UB	136.5 FIXED
\$200	47.2	1.7 LB	14.7 LB	84.3 UB	11,990 OPT	22.2 UB	136.5 FIXED
\$-50	44.4	1.7 LB	14.7 LB	61.7 OPT	7,940 OPT	22.2 UB	136.5 FIXED
<u>$X_{14} = 5.87$</u>							
\$0	45.7	1.7 LB	14.7 LB	70.1 OPT	9,670 OPT	22.2 UB	136.5 FIXED
\$100	48.1	1.7 LB	14.7 LB	82.1 OPT	10,640 OPT	22.2 UB	136.5 FIXED
\$200	49.7	1.7 LB	14.7 LB	84.3 UB	12,040 OPT	22.2 UB	136.5 FIXED
\$-50	43.8	1.7 LB	14.7 LB	60.0 OPT	8,850 OPT	22.2 UB	136.5 FIXED

^a

LB - Lower bound of the variable

UB - Upper bound of the variable

OPT - Optimal value of the variable (LB < OPT < UB)

^b X_{19} was manipulated, but the upper bound was fixed at 136.5

less for the second \$100. This is so, because x_{16} reached its maximum value sometime during the second \$100 increase. With only salary left to make a contribution, the rate of increase became far less.

This suggests a practical problem in incrementing the budget. When highly contributing variables reach their maximum values what would happen if extrapolation were made beyond the observed range of values? Given that x_{16} (ADPREP) makes a major contribution, it could be that extending x_{16} beyond its observed maximum (84.3) would continue to result in similar contributions. Of course, such analysis would have to be regarded as highly speculative.

Perhaps the budget increments that were chosen were too large. It may be unlikely that a school district would increase its per-pupil budget by \$100 in a given year. Given a more moderate increase where chief variables do not attain maximum values could generate a new population of values. Those districts that were previously near the observed maximum would use the budget increment to increase values of x_{16} beyond the previous maximum value. With a new population of values, another production function analysis would establish parameters based on the new population. Essentially, then the process of estimating a production function can be conceived as iterative in

nature. Each new year provides additional information by which the analysis can be revised and extended.

CHAPTER IV

SUMMARY, DISCUSSION AND SUGGESTIONS FOR FUTURE STUDIES

Summary

Educational decision-makers are faced with the task of allocating school resources in such a way as to maximize student outcomes. The problem of how best to proceed is complicated by the question of priorities among the outcomes to be attained, and because of the multiplicity of student outcomes, it is not likely that all outcomes can be maximized at once. Also, aside from questions of technical efficiency, decision-makers are constrained in their choice of plans by a limited budget and by social, political, and legal forces. Thus, the process of maximizing outcomes is irrevocably placed in the context of planning the best available use of scarce resources. The desire for rational decision-making entails that decision-makers have some empirical notions of how the various inputs into the educational process relate to student outputs. The prices of school inputs must be known if the question of allocative efficiency is to be considered.

Given this background, the rationale for the present study was a response to the need for studies that demonstrate the utility and limitations of mathematical models as aids in improving educational planning and decision-making. The problem of the study was to develop a mathematical model to facilitate the decision-making process in selected areas of educational activity by optimizing the allocation of scarce resources and to empirically illustrate its application. In development of the model attention was given to

1. Determining the educational production function that describes the input-output relationship between selected variables,

2. Determining the optimal combination of inputs to maximize output subject to certain costs and other constraints,

3. Determining the optimal combination of inputs to maximize output, subject to certain constraints, given increments in the budget constraint.

To accomplish the above first required that the process of education be conceptualized. Mathematical models do not evolve in the absence of a conceptual framework. Basically, the process of education was viewed as a function of the various inputs into the process. These inputs include student, community, and school factors. The

first two are regarded as essential to the conceptualization of the process, as these factors necessarily interact with, indeed may be causal determinants of, the various school factors. Given this interaction among the various input factors together with presumed causal directions, the process of education results in certain outcomes. Among these are cognitive achievement, affective growth, and physical development.

With this conceptual framework, the translation to a mathematical model occurred. Concurrent with this translation was the desire to allocate school resources in such a way that would maximize certain student outcomes. Thus a mathematical model was first required to estimate the educational production function and the budget equation. Given the production function as an objective function and the budget equation as one of the constraints, an optimization strategy was employed. Other constraints can be based on empirical evidence or the subjective desires of decision-makers. Also, the constraints should include minimum standards to be achieved by other outputs and bounds on the observed variation in the independent variables.

To estimate the production function and the budget equation, cross-sectional data were used in a survey-type design. The data consisted of aggregate measures on input

factors in 181 local school districts in a given state. Multiple regression techniques were used to estimate the parameters of the production function and the budget equation. The optimization strategy employed the techniques of mathematical programming, with the production function as an objective function and the budget equation as one of the constraints.

The school inputs used in the production function described only a limited area of educational activity. In particular, certain teachers characteristics were considered: experience, training, and level of salary. Also, classroom size, pupil-support personnel ratio, and the size of the district were used. Given the various inputs (i.e., student, community, and school) into the educational process, the following non-school inputs were found to make significant contributions in explaining the variation in reading achievement at the sixth-grade level (x_{21}):

1. Percentage of gross incomes over \$10,000 in the school district (x_3).
2. Attendance: ratio of ADA to ADM (x_6).
3. Percentage of graduates receiving post high school education or training (x_7).

Also, the following interactions of non-school inputs with school inputs were found to be significant:

4. Interaction of the percentage of ESEA Title I students with the pupil-support personnel ratio (x_4x_{19}).

5. Interaction of the percentage of minority enrollment with average class size (x_5x_{18}).
School inputs were forced into the regression equation since estimates of the effect of each were needed. The above led to the estimation of a production function with an $R^2 = 0.79$.

In estimating the budget equation, per-pupil instructional expenditures (x_{24}) was used as the dependent variable. School variables were forced into the equation so that the prices of each could be estimated. Only one non-school variable made a significant contribution, namely the percentage of minority enrollment (x_5). Curiously, it was the square of this factor (x_5^2) which entered the equation. This states that as the percentage of minority enrollment increased, per-pupil cost increased quadratically rather than linearly. Two school variables had significant square terms, namely percentage of teachers with advanced preparation (x_{16}) and median teacher salary (x_{17}). Inclusion of transportation factors (x_{12} and x_{20}) significantly increased the predictive power of the model. The budget equation was estimated with an $R^2 = 0.94$.

Given the production function and the budget equation together with bounds on the observed variation in the independent variables, a mathematical programming problem was formulated. For meaningful analysis, only school variables x_{13} , x_{15} , . . . , x_{19} were manipulated at various levels of x_{14} and x_{22} . Two levels of x_{14} (percentage of teachers with less than 4 years training) were considered, $x_{14} = 0$ and $x_{14} = 5.87$ (population mean). Some practical difficulties in using $x_{14} = 0$ occurred due to the indicated bounds on the variables. The clearest results emerged when $x_{14} = 5.87$. At this level, it was found that by re-allocating existing resources and maintaining a minimum standard for pupil-support personnel ratio ($x_{19} = 136.5$: population mean) that the predicted increase in achievement (x_{21}) that would result was 0.38 standard deviations. The analysis suggested that by eliminating salary increments due to experience, by increasing classroom size to the maximum observed, enough resources are released to increase the percentage of teachers with advanced training (x_{16}) by 17.6 percent and to increase the median salary (x_{17}) \$520 above that due to increases in x_{16} . Given this level of efficiency, if the per-pupil budget were incremented by \$100, the predicted increase in achievement would be 0.35 standard deviations. Given the existing allocation of school

resources, if a \$100 increment were made so as to maximize achievement, the predicted increase would be 0.73 standard deviations.

It was found that increases in district size produced only marginal increases in achievement. However, if district size were increased from 9,180 (population mean) to 50,000, enough resources would be released to increase x_{16} by 2.1 percent and x_{17} by \$170 above that due to x_{16} .

Throughout the analysis it was apparent that the percentage of teachers with advanced training (x_{16}) was the dominant effect followed by salary (x_{17}). The importance of salary was a function of the percentage of teachers with less than four years training (x_{14}). The greater x_{14} was, the greater the effect of salary. When $x_{14} = 0$, the effect of salary was minor compared to the effect of advanced training.

Discussion

Given that the primary focus of the study was to develop a mathematical model to aid decision-makers in educational planning, the most relevant concern is whether or not the model actually worked under real conditions. If the results suggested in the analysis for the 181 school districts involved were actually implemented, would changes

in achievement be within a reasonable range of that which was predicted? Of course, the answer to that question is presently unanswerable. The only way the adequacy of the model could be known was to actually implement the policies suggested by the analysis. However, a one-shot case approach could not constitute grounds for adequate verification. To prove the adequacy of the model would have required that it be tested for a sufficiently large number of cases and then consider the ratio of successes to failures, such ratio being the ultimate criterion of effectiveness. Thus, if model forecasts were shown to be reasonably correct 95 percent of the time, then it could be stated that the model has been able to describe and predict reality very well, and could be expected to do so in the near future.

The primary limitation of the present study was in the research design. The major difficulty in a survey study is that causal significance can never be attached to the conclusions. Thus in the present study differences in achievement could be estimated among schools having different percentages of teachers with advanced training. What could not be known was whether actually increasing the percentage for given schools would produce the same differences. To establish such cause and effect relationships generally requires an experimental, longitudinal study

design. To find out what happens to a system when you interfere with it, you have to interfere with it, not just passively observe it.

In the present study it was recognized that omission of relevant variables can bias the estimates of the regression coefficients. Given the nature of a survey study, it can not be known with certainty if all influences acting on the dependent variables through the independent variables have been brought into the analysis. Nevertheless, merely pointing to this weakness does not discredit the study. One needs to show that a bias exists and that it matters. In the context of the conceptual framework it is important to recognize the nature and extent of controls that were used in the empirical application. For instance, an attempt was made to control for student background through the use of SES variables (percent of incomes above \$10,000); for community influences through the use of such variables as attendance and future training; and for student peer-group effects through percentage of minority enrollment and ESEA Title I pupils. No doubt, other influences were omitted from the model, but to what extent the coefficients were biased is not clear. It could be that given the present controls, remaining sources of bias may only slightly effect the coefficients.

Given this state of affairs, the chief contribution of the present study can be viewed as analytical. Analytical studies of this type are those that typically search for the most important relationships among the inputs of the educational system and its outputs, the principal purpose being to locate possibilities for improvement that appear worth exploring. The findings of such analyses are necessarily probabilistic. Even so, statements about "What would happen if . . . ?" are likely to lead to far wiser decisions about educational policies than the kinds of uninformed hunches on which educational decision-makers often rely. Theoretically, there is every reason to suppose that the type of analysis displayed in the present study should be highly useful in helping educators obtain some idea of how best to deploy available funds, facilities, and personnel so as to maximize the educational outcomes students will attain, or to arrive at informed judgments about what trade-offs might be made among several kinds of inputs.

Perhaps one of the more important conclusions emerges from the analysis of real data used to illustrate the model. Of the school effects studies that have taken place, one of the most frequent conclusions has been that because of the relatively small regression coefficients of school variables

as compared to community and student variables, no proposed reallocation among present school variables could affect achievement very much. Smith, in his reanalysis of the Coleman study, described the situation as follows:

I think that we have now milked the EEOS data dry with respect to the 'determinants' of verbal achievement. The overall results should be clear to policy-makers and researchers alike. With regard to the differences among schools in resources that we conventionally measure and consider in making policy, there are few that give us any leverage over students' achievement. Within the fairly broad boundaries of existing variations, the simple manipulation of per-pupil expenditure or the hiring of more experienced teachers or the instituting of a new curriculum does not lead to dramatic changes in students' verbal achievement. The myth that the reallocation of conventional inputs will lead to a redistribution of achievement outputs can no longer be accepted.¹

Yet, the results of analysis in the empirical application suggested that by reallocating existing school resources so as to maximize achievement, a significant increase would result. Furthermore, incrementing the per-pupil budget by \$100 led to a predicted increase in mean achievement of 0.73 standard deviations. But, it was also true that the regression coefficients of school variables in the analysis were small compared to other inputs. This appears to

¹Marshall S. Smith, "Equality of Educational Opportunity: The Basic Findings Reconsidered," On Equality of Educational Opportunity, p. 395.

describe a situation where mathematicians are prone to lament, "Intuition can lead one astray." For it is one thing to look simply at regression coefficients and to speculate at the effects of reallocating resources and another to estimate the prices of these resources and to specifically employ an analysis which seeks to maximize achievement within the confines of a given budget.

Suggestions for Future Studies

Better Data and Variables

The primary goal of the study was to develop a model. To illustrate the utility of the model, reference to real data was made. While the data that were used could by no means be considered an ideal set, the purpose of illustrating the model was served. No doubt, future application of this and similar models should require a more comprehensive set of data and variables. A list of potential variables that could find their place in such a model is given in the appendix. To avoid problems in interpretation it is suggested that future studies use mean teacher salary as a variable rather than the median, and that the absolute numbers as well as percentages of the teacher variables be collected. In the present study it was not possible to indicate what the final median salary would be as the result

of a reallocation. This results from the fact that the median is an ordinal measure. However, given the mean and the absolute numbers for each teacher variable, this problem could be resolved.

It is also felt that a cost of living index is needed to avoid biasing the parameters of the budget equation. It may be that the absence of this variable does not seriously effect the estimation of the parameters, but it cannot be known for sure until such a variable has been brought explicitly into the analysis.

Bayesian Procedures

One of the shortcomings observed in the present analysis was found when the budget constraint was incremented. It was found that very quickly one reached the bounds of observed variation in relevant variables, analysis being inhibited beyond that point. At the same time, it is recognized that commitment to the modeling process in education does not mean a one-shot survey but a periodic reassessment. Thus the production function should be estimated annually. To do so leads to two distinct advantages. First, with each new year to be considered, variation in the ranges of variables previously observed is expected to occur. Also, if results of previous analyses have been implemented, then wider ranges are to be expected in the

important variables. If such a variable has previously been deployed to its maximum value, then a new population presents a new range of variation and analysis proceeds.

More importantly, with periodic assessment, one can proceed in a manner analogous to Bayesian statistical procedures. This means that prior information based on previous years' analyses can be used to improve and perfect current estimation of the regression coefficients of the various input factors. Given that cyclical variations may occur from year to year, the process is deemed important for determining the average effects of variables over a period of years.²

Technical Efficiency

While the present study was limited to the investigation of allocative efficiency, the question of technical efficiency should not be avoided. In a previous chapter methods of investigating technical efficiency have been considered. In particular, it is suggested that the residuals of production function analysis for this and other studies be examined closely. An experimental approach should be employed in ferreting out additional variables

²Donald L. Meyer, "Bayesian Statistics," Review of Educational Research, XXXVI (December, 1966), 503-516.

that can lead to new technologies in education. For all districts that essentially have equivalent combinations of inputs, the residual reveals the most productive districts in the group. Comparisons of these districts with other districts in the group on the types of programs, staff, and facilities lead to technical improvements.

Causality and Policy Implications

Even though the results of the analysis in the present study were not statements of causality, it is clear that if decision-makers were expected to implement the results of the analysis causal significance would be implicitly assumed. Social policy research must always contend with this dilemma. As a consequence methodological approaches to this problem are more and more being considered. Blalock has been a strong leader in this area and has contributed much to its development.³ The most relevant term to describe this approach is path analysis. It involves a type of regression analysis called "causal". One postulates various alternative causal schemes for explaining a phenomenon under investigation. The analysis eliminates less plausible causal schemes. Of course, causality can

³Blalock. See also, Blalock, Methodology in Social Research (New York: McGraw-Hill, 1968).

never empirically be determined with exactitude, but the analysis can narrow the field of possible alternatives. And it is precisely this fact with which policy research must contend. It is the regression coefficients that give the laws of science. Educators must begin to make statements of the form, "A unit increase in x is believed to cause b units increase in y ." Correlations are of limited value to policy makers. They need to know the change expected in altering the status quo. Regression coefficients can give this information.

Beyond Cognitive Achievement

In thinking about how much can be accomplished by improvements in education it is necessary to go outside the internal system of school achievement, especially cognitive achievement, to consider the possible size of long-run effects of improvements in education for occupation and income for those who have been deprived. It has been stated that previous studies show that academic achievement is not necessarily related to future success in life. Yet the realization of the American dream is for most realized in the dual category of occupation and income. For maximizing both the goals of school and society it would be well to look at the relationship of various school inputs

to future occupation and income. Such research may be in the distant future, but it is a goal that should not be denied.

More Empirical Studies

It is hardly to be questioned that studies confined to the practical applications of models should be energetically pursued to provide, eventually, a firmer basis than now exists for dealing with broad questions of educational policy. Mathematical models, such as was developed in this study, allow decision-makers to form expectations of future consequences, these expectations being based on known empirical relationships and the decision-makers' judgments.

All of the foregoing does not mean that in applied work the researcher even pretends to be able to find the best of all possible decisions. The data are too incomplete and sometimes inaccurate, the tools of analysis are often too blunt, and the researchers knowledge of the educational process is too limited for him to be able to come up with anything more than approximations to the ideal of the true optimum. Nevertheless, an analysis which is specifically designed to look for optimal decisions, crude and approximate though it may be, is very likely to do much better than the workable but relatively arbitrary rules of thumb which play so prominent a part in educational practice.

APPENDIX

A LIST OF POTENTIAL VARIABLES
FOR ESTIMATING THE
EDUCATIONAL PRODUCTION FUNCTION

I. Pupil Characteristics

Ethnic origins (percentages)

Percentage of transiency

Ratio of aggregate attendance to aggregate
membership

Percentage of children receiving free lunch

Percentage of vocational students

II. Community Characteristics

Adjusted gross income per pupil

Median family income

Median years of schooling in adult population

Percentage of adults in unskilled, semi-skilled,
and service occupations

Percentage of children in two-parent families

Percentage of uncrowded housing units

Percentage of children 5-17 enrolled in public
schools

Percentage of non-white population

III. School Variables

A. Teacher variables

Average class size

Pupil-teacher ratio

Teacher experience

Teacher salaries

Teachers' verbal ability

Percentage of teacher turn-over

Teachers academic preparation (college hours
in subject taught)

Number of instructional assignments per teacher

Percentage of teachers on tenure

B. Administration variables

Number of professional supportive staff per
1,000 pupils

Median administrative salary

Subject matter offerings

Length of school year

Ability grouping

Textbook supply

District enrollment

Per-pupil paraprofessional ratio

Number of full-time employees in auxiliary
services per pupil

C. Facilities

Number of library books

Age of building

Average size of high schools

School site size

Percentage of makeshift classrooms (portables,
etc.) per 1,000 students

Percentage of pupils on double session

Science laboratory facilities

D. Expenditures

Instructional expenditures per pupil

Administrative expenditures per pupil

Total expenditures per pupil

Expenditures on instructional materials and
supplies per pupil

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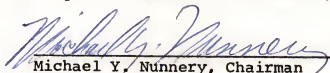
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BIOGRAPHICAL SKETCH

Jackson K. McAfee was born December 27, 1939, at Vero Beach, Florida. In June, 1957, he was graduated from Vero Beach High School. He received the degree of Bachelor of Arts from North Park College In June, 1961, with a major in mathematics. From September, 1961, to June, 1963, he taught mathematics at North Park Academy, Chicago, Illinois. He received the degree of Master of Arts in mathematics from Northwestern University in August, 1965. In 1967 he attended a National Science Foundation Summer Institute in Computer Science and Related Mathematics at the University of Oklahoma. He taught mathematics at the Vero Beach Senior High School from September, 1963, to June, 1970. In September, 1970, he entered the University of Florida to work toward the degree of Doctor of Education.

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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Education.



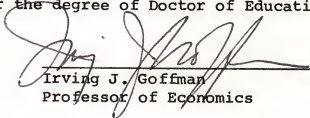
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Professor of Educational
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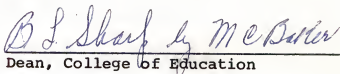
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August, 1972



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